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# *EVIDENCE OF STREAM MOTION AFFORDED BY THE FAINT STARS NEAR THE ORION NEBULA*

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The increased accuracy which can be obtained in the determination of proper motions of the stars by long-focus instruments makes it necessary to take into account quantities which otherwise could be neglected. When the probable error of the motion in a coordinate becomes as small as 0".003 or 0".004, we may no longer neglect systematic errors of this size or larger, if they can possibly be determined.

In a discussion of the proper motions of 85 stars in the region of the Pleiades,[1] photographed with the 80-foot focus of the 60-inch Mount Wilson reflector it was found that in the reductions the quadratic terms of the coordinates could not be neglected, notwithstanding the fact that the size of the field was only 24 by 30 minutes of arc. For this and other reasons it was suspected that by neglecting such terms in a former discussion of the proper motions of 162 stars near the Orion nebula,[2] photographed with the 40-inch Yerkes refractor, we had not attained the best possible results. A new solution has therefore been made including the quadratic terms; the necessary corrections are of the same order as those for the Pleiades field and may not be neglected. The comparison reveals the relative efficiency for this kind of work of the long-focus reflector, which seems easily able to withstand a comparison with the refractor.

The fact that we are here dealing with a field in which many of the stars must belong to the nebula, especially in the center of the plates, has its disadvantages, as well as its advantages, for, the motion of the nebula being small, we cannot separate by their proper motion alone the individual stars belonging to the nebular system. This necessitates the exclusion for reference purposes of most of the central stars, whereby the weight of the reductions is considerably lessened.

On the other hand, from the abundance of stars near the Trapezium we can conclude that most of them form a part of the system of the Orion nebula. From their connection with the nebulosity other stars can also be selected as probable members of the group. Except for any internal motion, which, however, can be neglected in a first approximation, all these stars must have the same proper motion; they therefore enable us to drive any magnitude error in case such error does exist.

Another advantage of a field like that discussed here, is that, although the small field measured contains only two or three stars for which meridian observations are available, we can now use for the reduction of our relative to absolute motion all the stars in the catalogues which seem to belong to the system of the Orion nebula. For this purpose twelve stars in Boss's *Preliminary General Catalogue* were found to be available. The motions previously derived were corrected in this way and reduced to absolute motions in Boss's system. The results for different groups of stars are as follows:

13 stars which seem to be enveloped in nebulosity have

$$\mu_\alpha = +0''.0052 \pm 0''.0020$$
$$\mu_\delta = -0''.0024 \pm 0.0024$$

32 stars near the Trapezium have

$$\mu_\alpha = +0''.0059 \pm 0.0013$$
$$\mu_\delta = -0.0021 \pm 0.0010$$

21 stars, found to be variable by different observers,

$$\mu_\alpha = +0.0046 \pm 0''.0009$$
$$\mu_\delta = +0''.0011 \pm 0''.0011$$

Excluding here three stars for which the variability is uncertain, we find

$$\mu_\alpha = +0.0049 \pm 0''.0009$$
$$\mu_\delta = -0''.0003 \pm 0''.0010$$

From this it is clear that practically all these variables must belong to the system of the Orion nebula and that their variability is therefore due to a physical connection with the nebula.

From the work of Buisson, Fabry and Bourget,[3] we know there is some evidence that the nebula rotates about an axis NW − SE, the NE portion receding, the SW approaching. As their measures are confined to the region within 2′ from the Trapezium, I have investigated whether the stars measured within the same area show the effect of such a rotation. As most of the stars seen in the densest part are likely to be on this side of the nebula, they ought to show a preponderance of motion from SW towards NE. The mean motion in this direction (27 stars) was found to be $0''.0010 \pm 0''.0012$, while the mean motion at right angles is $0''.0000 \pm 0''.0015$. This small systematic motion is well within the limits of the probable error.

For the background stars, that is, all stars not included in the other groups, the change in the mean values of $\mu_\alpha$ and $\mu_\delta$ with magnitude, and the fair agree-

ment of these values with the parallactic motion, prove that we have been successful in excluding from this group most of the stars which are connected with the nebulosity. As a further test I have constructed the diagrams in Fig. 1. The number of stars and the total motion were determined for sectors of 15°; then to smooth the curves, overlapping means for three successive sectors were formed. In order to avoid the influence of the large proper motions, stars with $\mu > 0''.050$ were excluded. The smaller diagram shows the results for the number of stars, the larger for the total motion in the different directions. If among the background stars we have included a considerable number of nebula stars, we should find a preponderance of both numbers and motion in the direction of $p = 110°$, in which the nebula is moving; the fact

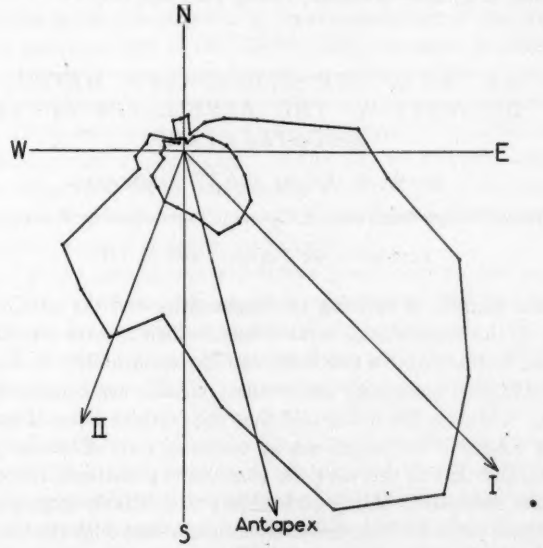


FIG. 1

that no such preponderance appears, is another proof that only a few stars of the nebula system can have been included among the background stars.

The same diagram shows that neither the maximum frequency nor the maximum motion coincides with the direction of the parallactic motion. To investigate whether this fact can be accounted for by stream-motion, I computed the direction of both streams, adopting as coördinates of the vertices:

Stream I, $\alpha = 91°$, $\delta = -15°$
Stream II, $\alpha = 288°$, $\delta = -64°$

We find for

Stream I, $p = 139°$, $\lambda = 13°$
Stream II, $p = 191°$, $\lambda = 108°$

The agreement of the maximum frequency as well as that of the maximum motion with Stream I is quite remarkable, when we take into account the small number of stars available (88), the more so as λ is only 13°; the influence of Stream II also, although not so strong, can be plainly seen.

More material for different parts of the sky will of course be necessary before we can conclude with any certainty that stars as faint as the 14th and 15th magnitudes show the stream-motion.

[1] van Maanen, Adriaan, *Mt. Wilson Contr.*, No. 167, 1919.

[2] van Maanen, Adriaan, *Astr. J., Albany, N. Y.*, 27, 1912 (139–146).

[3] Buisson, H., Ch. Fabry, and H. Bourget, *Astrophys. J., Chicago, Ill.*, 40, 1914 (241–258).

[4] Eddington, A. S., *Stellar Movements, London*, 1914, page 100.

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## *ON THE USE OF THE SPECTROSCOPIC METHOD FOR DETERMINING THE PARALLAXES OF THE BRIGHTER STARS*

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When the method of deriving the luminosities and the parallaxes of stars by means of the intensities of certain lines in their spectra was developed by Adams and Kohlschütter a few years ago the applicability of the method to the stars of highest luminosity and smallest parallax was necessarily somewhat uncertain. This was due to the fact that the method depends upon the calibration of a scale of line-intensities by means of stars of known parallax and magnitude, and that at this time the observational material for stars of small parallax was necessarily scanty and subject to relatively large percentage errors. All that could be done was to select a few stars with the best parallaxes, so far as could be estimated, and base upon them a set of provisional reduction-curves for the spectroscopic determinations. As a result the values for the absolute magnitudes derived in this way while sufficiently accurate to indicate clearly that the parallaxes of certain stars were very small were not of such a quality as to show in all cases the slight differences between individual stars of small parallax.

The situation has improved greatly in recent years. On the one hand the parallaxes of a large number of stars have been measured with high accuracy by various observers with the aid of photographic methods. On the other hand the amount of spectroscopic material for the stars of various magnitudes and proper motions has accumulated to such an extent that use can be made of the extremely valuable method of determining mean parallaxes for groups of stars from the parallactic motion. Accordingly it has now become possible

to subject the spectroscopic method to a rigorous test of its applicability to the stars of high luminosity and small parallax. The importance of this question is evident when we consider that the trigonometric method of measurement with a probable error which is nearly independent of the size of the parallax can give but approximate values of the absolute magnitude for stars of very small parallax. Thus if the measured parallax of a star of the fifth magnitude is $+0''.010 \pm 0''.005$ its absolute magnitude may lie anywhere within the limits $-1.5$ and $+0.9$, and for smaller parallaxes the range is much larger. Since a great majority of the brighter stars in the sky have small parallaxes the use of the spectroscopic method in which the error is more nearly proportional to the size of the parallax is of fundamental importance for such cases.

As a first step in the investigation a new determination of the constants of reduction for the intensities of the spectral lines was made in 1918 with the aid of the large number of measured parallaxes then available. The absolute magnitudes based upon this system have been divided into groups and comparisons have been instituted with the values obtained from parallactic motion and all available measured parallaxes. In this way we can determine whether the absolute magnitude varies continuously with the line-intensity; and it is possible to use a large number of measured parallaxes including stars of different proper motion and apparent magnitude.

The results of the comparison are shown graphically in the accompanying figures. The spectroscopic absolute magnitudes $M_s$ are plotted as abscissae, the line-intensities being indicated on parallel axes. The absolute magnitudes $M_c$ derived from measured parallaxes and from parallactic motion are plotted as ordinates. The full-line curve represents the values computed from parallactic motion; the broken curve those from measured parallaxes. The wide break in the curves for the $K_4$–$K_9$ and the M stars are due to the well-known division of these stars into the giant and dwarf classes.

It is at once evident that the computed absolute magnitudes vary continuously with the spectroscopic magnitudes quite as well among the stars of highest luminosity as among the fainter stars. In fact the largest discordance is found not among the brighter stars but among some of the faintest stars of groups $A_7$ to $F_8$ where the luminosities as computed from measured parallaxes show but little variation with line-intensity and for the very faintest stars appear to act in the wrong direction. It is to be noted that in this comparison the more accurate result for the more distant stars is given by the parallactic motion and for the nearer stars by the measured parallaxes.

It is clear that if there are no systematic corrections to be applied to the spectroscopic absolute magnitudes used in this comparison the curves should consist of straight lines with an inclination of 45°. The results show that these corrections are relatively small and that the spectroscopic criteria hold throughout the range of magnitude investigated.

It should be realized fully that the method of deriving parallaxes from the intensities of the spectral lines depends for its accuracy upon our knowledge of the parallaxes of the stars used for the determination of the relationship between absolute magnitude and line-intensity. It is a process of approximation in the early stages of which the corrections may be considerable, but which

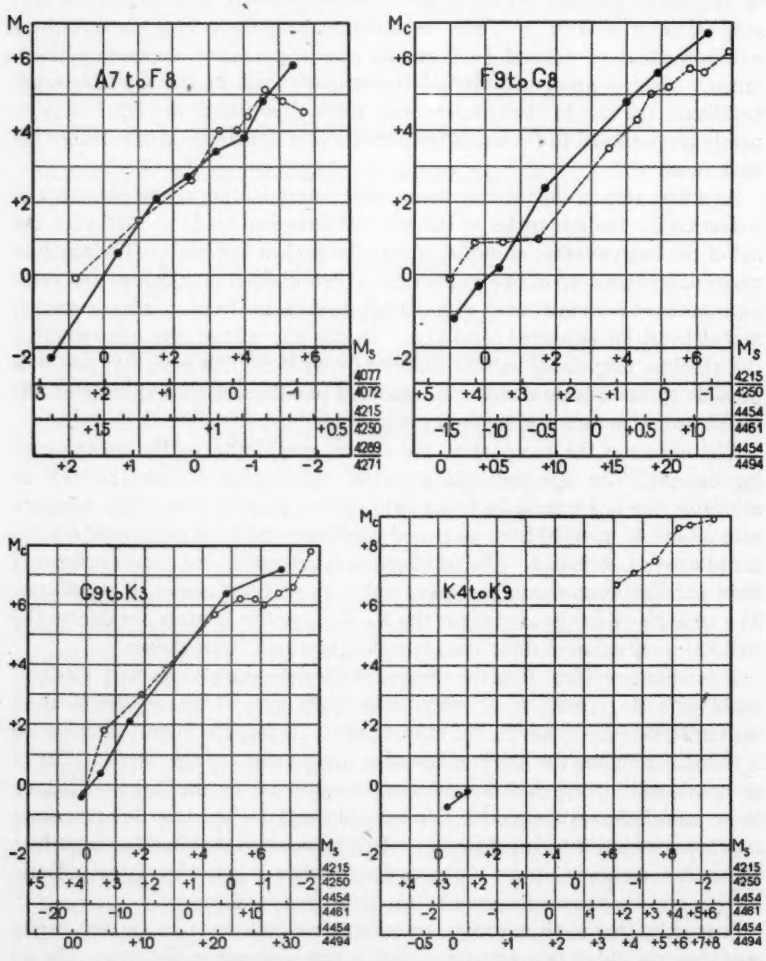


FIG. 1. CURVES SHOWING THE RELATIONSHIP OF THE SPECTROSCOPIC ABSOLUTE MAGNITUDES ($M_s$) TO THE MAGNITUDES AS DERIVED FROM PARALLACTIC MOTION AND MEASURED PARALLAXES ($M_c$)

The line intensities upon which the spectroscopic magnitudes are based are also given as abscissæ.

gains more and more in accuracy as the material upon which it is based becomes larger and more reliable.

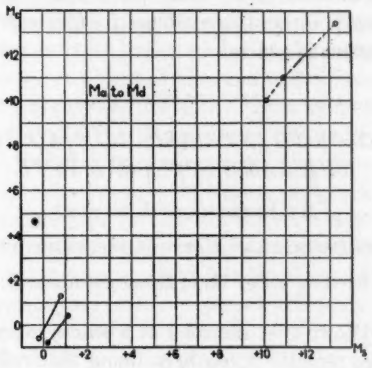


FIG. 2

To illustrate the present state of agreement between the spectroscopic and the trigonometric parallaxes the following table is added for purposes of comparison. The stars are divided into groups and the limits of proper motion for each group are given under the heading $\mu$. The symbols, $\bar{M}$ and $\bar{\pi}$ represent the average absolute magnitude and parallax, respectively. The last column of the table shows the differences between the spectroscopic and the trigonometric parallaxes. The agreement may certainly be regarded as satisfactory in view of the number of stars involved.

*Apparent Magnitudes 3.5 to 6.5*

| $\mu$ | No. | $\bar{M}$ | $\bar{\pi}_{spec.}$ | $\bar{\pi}_{trig.}$ | $\bar{\pi}_s - \bar{\pi}_t$ |
|---|---|---|---|---|---|
| $\leqq 0''.020$ | 23 | −0.4 | $0''.009$ | $+0''.008$ | $+0''.001$ |
| $0''.021$–$0''.040$ | 19 | +0.2 | 0.013 | 0.007 | +0.006 |
| 0.041–0.070 | 36 | +0.3 | 0.013 | 0.016 | −0.003 |
| 0.071–0.100 | 29 | +0.7 | 0.016 | 0.021 | −0.005 |
| 0.101–0.150 | 38 | +0.8 | 0.021 | 0.026 | −0.005 |
| 0.151–0.200 | 31 | +2.3 | 0.039 | 0.038 | +0.001 |
| 0.201–0.300 | 35 | +2.7 | 0.042 | 0.043 | −0.001 |
| 0.300–0.500 | 51 | +3.2 | 0.042 | 0.042 | 0.000 |

In conclusion a few words may be added about the extent to which the spectroscopic method of determining parallaxes has been applied at Mount Wilson. Between 1500 and 1600 stars have now been investigated in this way including nearly all stars with trigonometric parallaxes and an extensive list of those with very large and very small proper motions. It is of interest to note that in this large amount of observational material hardly a single serious contra-

diction has been found between the spectroscopic and the trigonometric results. This is the more remarkable since we should expect occasional idiosyncracies in the spectra of stars subject to exceptional physical conditions, and the rareness of such cases is interesting evidence for the uniformity of the development of stellar spectra in general.

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## *RELATION OF COLOR TO INTRINSIC LUMINOSITY IN STARS OF THE SAME SPECTRAL TYPE*

By Frederick H. Seares

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It has long been known that the color of a star is intimately related to its spectral type. More recently it has been found that color is also correlated with intrinsic brightness.[1] Thus, two stars having different luminosities, but the same spectrum so far as the usual criteria are concerned, may differ in color-index by more than half a magnitude.[2]

The phenomenon is perhaps to be accounted for as follows: Similarity in spectrum implies at least approximate equality in surface brightness. Difference in luminosity is therefore mainly a matter of size, the brighter object being the larger. But inequality of dimensions doubtless entails differences in atmospheric constitution, and hence also in the selective absorption occurring in the atmospheres. Since color is determined by the distribution of intensity in the continuous spectrum, while type relates more particularly to the characteristics of the spectral lines, one star may thus appear redder or bluer than another, although both are of the same type.

But whatever the explanation, an extension of the results of Adams, van Rhijn, Monk, and others to include a wide range of luminosity for each spectral type is immediately desirable, because of their bearing upon the difficult problem of stellar constitution. Further, any phenomenon correlated with stellar luminosity should be studied to the utmost, on account of its possible importance for the determination of interstellar distances. If the intrinsic brightness of a star can be found, the problem of its parallax is solved.

The method employed for the measurement of color in securing the preliminary results here described is that of exposure-ratios[3], which is convenient and reliable, and much more expeditious than the direct determination of color-index.

An isochromatic plate exposed behind a yellow filter records the intensity of the 'yellow' light received from the star. The same plate used without filter registers the 'blue' light; the longer wave-lengths are then also active, but the image, nevertheless, is essentially blue, owing to the relatively great blue-sensitiveness of the isochromatic plate. In neither case are we dealing with monochromatic light, or even with a very narrow range of color. That, how-

ever, is of no great consequence; it is only important that the mean wavelengths for 'blue' and 'yellow' should differ by a considerable amount. The relative intensity for the two spectral regions will then vary with the color of the star, and in consequence may be used as an index of its color.

Practically, however, it is simpler to use as a measure of color the ratio of the exposure times which will yield blue and yellow images of equal intensity, or, better still, the logarithm of this ratio. Thus we make a series of exposures to blue, usually $2^s$, $4^s$ and $8^s$, and then one or more to yellow, such that the resulting images fall within the limits of size fixed by the first and last blue exposures. For an early-type star the yellow exposures are generally $32^s$ and $64^s$, for an intermediate type $16^s$ and $32^s$, and for a late type $8^s$ and $16^s$. The logarithm of the exposure-ratio for equal blue and yellow images is then estimated directly from the plate. With a normal emulsion the logarithms for the principal spectral types (ratio of blue to yellow for stars of zero absolute magnitude) are approximately as follows: Bo, 8.79; Ao, 8.94; Fo, 9.10; Go, 9.26; Ko, 9.44; Ma, 9.55.

Troublesome photographic difficulties are avoided by using images of constant size produced by a constant series of exposure times, compensation for differences in apparent magnitude being made by varying the aperture of the telescope. With the 60-inch reflector stars as faint as the eleventh magnitude can be observed with the exposures given above.

The photographic emulsion is changed as infrequently as possible, for every change necessitates a systematic correction, and indeed each plate usually requires a small correction. From five to ten stars are ordinarily observed on each plate; and, since the adopted value for a star is based on several plates, the measures themselves afford a means of reducing the individual plates to a homogeneous standard system of colors.

The measures require a correction for atmospheric extinction, but since all the observations on a plate are made at about the same zenith distance, this is assumed to be constant and is determined as a part of the systematic correction for the plate. Errors depending on seeing and atmospheric conditions generally also enter into the plate correction and are thus largely eliminated from the results.

The internal consistency under favorable conditions is illustrated by the accompanying observations of the two components of 61 Cygni which extend over an interval of about two years. The logarithms of the exposure-ratios for the two stars, which were photographed simultaneously, are in the first two columns of the table. The average deviation of the differences in the third column corresponds to about 0.08 magnitude, which indicates the uncertainty in a single determination of the difference in color of two stars on the same plate. This differential result, which is free from systematic plate-errors, is equivalent to an uncertainty of 0.05 or 0.06 magnitude in one measurement of the color of a single star, a quantity of the same order as the average deviations found directly from the measures of each of the two components,

whence we conclude that the plate-errors have been successfully eliminated. The complete observation of a star, including the setting of the telescope, requires seven or eight minutes.

*Exposure-ratio—Components of 61 Cygni*

| log $R_1$ | log $R_2$ | DIFFERENCES | $v_1$ | $v_2$ | $v_3$ |
|---|---|---|---|---|---|
| 9.39 | 9.50 | −0.11 | +0.03 | −0.02 | +0.05 |
| 9.41 | 9.47 | −0.06 | +0.01 | +0.01 | 0.00 |
| 9.43 | 9.45 | −0.02 | −0.01 | +0.03 | −0.04 |
| 9.39 | 9.50 | −0.11 | +0.03 | −0.02 | +0.05 |
| 9.43 | 9.45 | −0.02 | −0.01 | +0.03 | −0.04 |
| 9.42 | 9.48 | −0.06 | 0.00 | 0.00 | 0.00 |
| 9.42 | 9.47 | −0.05 | 0.00 | +0.01 | −0.01 |
| 9.42 | 9.47 | −0.05 | 0.00 | +0.01 | −0.01 |
| 9.44 | 9.45 | −0.01 | −0.02 | +0.03 | −0.05 |
| 9.40 | 9.48 | −0.08 | +0.02 | 0.00 | +0.02 |
| 9.40 | 9.49 | −0.09 | +0.02 | −0.01 | +0.03 |
| 9.39 | 9.50 | −0.11 | +0.03 | −0.02 | +0.05 |
| 9.41 | 9.49 | −0.08 | +0.01 | −0.01 | +0.02 |
| 9.39 | 9.49 | −0.10 | +0.03 | −0.01 | +0.04 |
| 9.48 | 9.51 | −0.03 | −0.06 | −0.03 | −0.03 |
| 9.44 | 9.50 | −0.06 | −0.02 | −0.02 | 0.00 |
| Means 9.416 | 9.481 | −0.065 | ±0.019 | ±0.016 | ±0.028 (log R) |
| 1.09 | 1.27 | −0.18 | ±0.053 | ±0.045 | ±0.078 (magnitude) |

Results are now available for about 150 stars whose spectral types and absolute magnitudes have been derived mainly from three sources: the extensive and valuable lists of spectroscopic parallaxes by Adams and Joy, which include types F, G, K, and M;[4] the A stars known to belong to the Taurus Stream;[5] and the B stars whose parallaxes have been determined by Kapteyn.[6]

The weight of the results is very unequal; some of the stars used as standards have been observed a dozen or more times, while for others only a single observation is available. The nature of the correlation between color and luminosity is nevertheless clearly enough indicated, although quantitatively it will be subject to considerable revision.

The discussions thus far undertaken are very simple. The stars having absolute magnitudes between +1 and −1 were separated fom the others and arranged according to spectral type. By plotting spectrum against exposure-ratio a curve was obtained (circles, fig. 1;) which shows the relation of these two factors for stars of zero absolute magnitude. The dwarf stars of the later spectral types, whose mean absolute magnitudes are 6 or fainter, were similarly treated, thus giving the second curve (points) shown in fig. 1.

For $M = 0$ the variation is nearly linear up to K5; but beyond this point there seems to be no further increase in color with advancing spectrum, the latest K's (K6) and the various sub-classes of the M stars all having approxi-

mately the same color. Further, the result noted by earlier observers is at once evident, namely, that the giant G and K stars are redder than the dwarfs of the same spectrum. The difference becomes zero for $F_5$, and is again zero, or at least very small, for the M stars, although the difference in luminosity between the giants and dwarfs is here about 10 magnitudes with a corresponding range in intensity of 1 to 10,000.

For further study of the relation of color to luminosity we consider groups of stars having the same spectral type. As they stand, the data are insufficient; but provisionally we proceed by reducing all the stars within any spectral sub-division to the mean spectrum with the aid of the curves of fig. 1. Thus all the B stars are reduced to B5 by applying to the logarithms of the

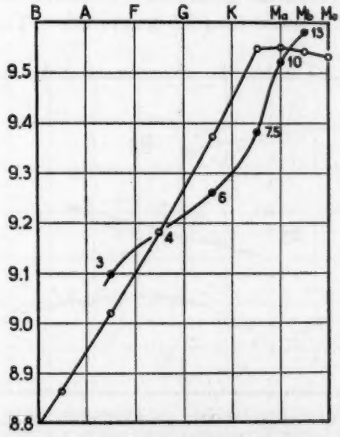


FIG. 1. ORDINATES: COLOR (LOGARITHM OF EXPOSURE-RATIO)
ABSCISSÆ: SPECTRAL TYPE

The open circles represent mean valves for stars of zero absolute magnitude. The dots refer to stars of fainter luminosity, the respective mean absolute magnitudes being indicated by the accompanying figures.

exposure-ratio of each a small correction which makes the colors very nearly what they would have been had the types actually been B5 throughout. The six groups of results thus obtained have been plotted separately—logarithms of exposure-ratios as ordinates against absolute magnitudes as abscissae —and are shown in figs. 2 and 3.

The results for the B stars are to be regarded as altogether provisional. They suggest that the color decreases, that is, becomes bluer, as the intrinsic brightness decreases. The A stars, all of which belong to the Taurus stream, increase in color with increasing absolute magnitude. This result seems definite, and is confirmed by the Fo and F1 stars derived from the lists of Adams and Joy, which are shown by the crosses in fig. 2. When reduced to A5 they agree well with the A stars of the Taurus stream. A further confirmation is

found in the fact that these A stars, whose absolute magnitudes, with one exception, lie within the range $M = +1.2$ to $+3.0$, agree with the curve for $M = 0$ determined by the adjacent spectral types (see fig. 1,) only after their colors have been reduced to $M = 0$ in accordance with the relation shown in fig. 2.

The giant and dwarf F stars in the mean have sensibly the same color, but when the individual results are plotted we find pretty definite evidence of a change in color as indicated in fig. 2, first an increase and then a decrease in color with increasing $M$, the maximum lying somewhere near $M = +2$.

The results for the G's and K's (fig. 3;) are qualitatively certain beyond any question, the giants as before remarked being very much redder than the dwarfs. For the K stars the difference in the corresponding color indicates for $M = -1$ and $+7$ is more than 0.6 magnitude. This large difference is

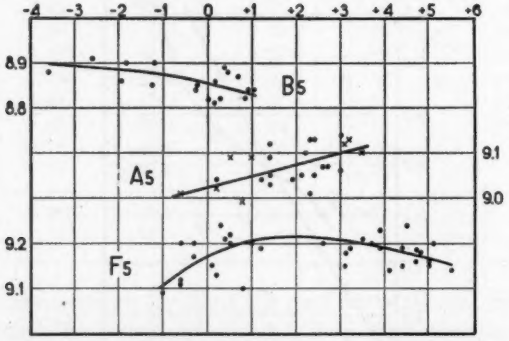


FIG. 2. VARIATION OF COLOR (LOGARITHM OF EXPOSURE-RATIO, AS ORDINATE) WITH ABSOLUTE MAGNITUDE (ABSCISSA) FOR B, A AND F STARS

The points represent individual stars whose colors have been reduced by the curves of Fig. 1 to the mean spectra, B5, A5, etc., for the respective classes. The crosses in the A5 group are F0 and F1 stars reduced to A5.

immediately suggestive of the errors that may occur in attempting to infer spectral type from color, or to determine visual magnitude from photographic with the aid of the spectral type, as is not infrequently done.

Among the dwarfs themselves, neither for the G's nor the K's, does there appear to be much change of color, although the faintest K's suggest that a minimum occurs at $M = +7$ and that from there on the color increases with increasing $M$.

The last curve of fig. 3 illustrates the similarity of color for the giant and dwarf M stars.

In spite of the diversity of the curves, relationships for adjacent spectral classes can usually be traced. Thus the curve for the A stars seems to be related to the ascending branch of that for the F's, while the descending branch of the latter curve reappears in the results for the G's and K's. The behavior

of the M's is peculiar, and owing to the lack of late K types the transition cannot certainly be traced. The interrelation of the B and A stars is also open to question. From the present results we should infer that for Ao stars there is little or no dependence of color upon absolute magnitude. This indeed seems to be the case. For example, Vega and the companion to $\sigma^2$ Eridani, both Ao and differing intrinsically by more than ten magnitudes, are very nearly of the same color.

The change in the relation of color to luminosity from type to type is so rapid that a closer subdivision of the spectral classes is desirable, but this must await the accumulation of further data.

Aside from the bearing of these results upon the problem of stellar constitution, which it would be premature to consider at present, it is perhaps worthy

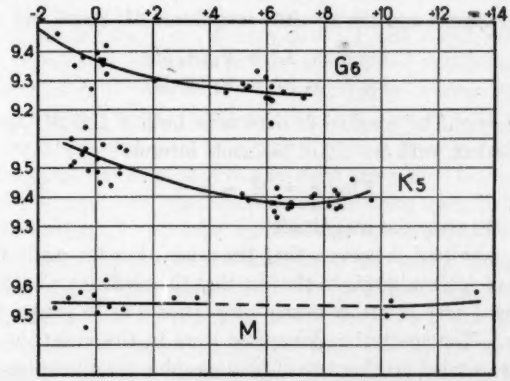


FIG. 3. VARIATION OF COLOR (LOGARITHM OF EXPOSURE-RATIO, AS ORDINATE WITH ABSOLUTE MAGNITUDE (ABSCISSA) FOR G, K, AND M STARS

All the G and K stars have been reduced in color G6 and K5, respectively. Ma. Mb, and Mc stars have been combined without correction for color.

of remark that they may occasionally be useful for determinations of distance Thus in the present case the results afford a useful control on the parallax of the Taurus Stream. As previously mentioned, it is only when reduced to zero absolute magnitude with the aid of the curve for the A5 stars that the colors of the Taurus stars are brought into agreement with the smooth curve for zero absolute magnitude defined by the adjacent spectral types (see fig. 1). Conversely, had the parallax of the Taurus Stream been unknown, it could have determined from the data now available with an indicated uncertainty of about 20 per cent, or $\pm 0''.005$, which is comparable with the precision of the result usually adopted.

The use of any such method, however, presupposes a knowledge of the spectral types of the stars in question. But if spectra are available, the spectroscopic method of determining distance can immediately be employed; at least

this will be true if stars of intermediate or late type are present. The results afforded by observations of color would however be useful as a control on spectroscopic or trigonometric determinations of parallax, although they might not add greatly to our knowledge of stellar distances. But if the method depending on color measurements could be extended in such a way as to obviate the necessity of direct spectral observations, the advance would be of importance, for it could then be applied to faint stars beyond the reach of both trigonometric and spectroscopic methods of measuring distance.

Theoretically such a procedure seems not impossible. Suppose, for example, that we observe three spectral regions instead of two, and that we combine the results in such a way as to yield two exposure-ratios, $\log R\ (\lambda_1, \lambda_2)$ and $\log R\ (\lambda_1, \lambda_3)$, $\lambda_1$, $\lambda_2$, and $\lambda_3$ representing the effective wave-lengths of the three regions in question. Presumably both these quantities would be functions of the spectral type $S$ and the absolute magnitude $M$; hence, we could write

$$\log R\ (\lambda_1, \lambda_2) = F_1\ (S, M)$$
$$\log R\ (\lambda_1, \lambda_3) = F_2\ (S, M)$$

from which it might be possible to determine both $S$ and $M$, and therefore, finally, the parallax, with the aid of the usual formula

$$5 \log \pi = M - m - 5$$

in which $m$ is the apparent magnitude.

It must be admitted, however, that the trials thus far made hold out no great promise of success, owing to the fact that the coefficients in the two equations involving $S$ and $M$ are so nearly proportional as to render the solution indeterminate. The spectral regions used were in the violet, blue, and yellow. These were tried first because of photographic considerations; but others may prove better adapted to the requirements of the problem, and it is proposed to give the question further attention.

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[2] Seares, F. H., loc. cit.

[3] Seares, F. H., These PROCEEDINGS, **2**, 1916 (521–525).

[4] Adams, W. S., and A. H. Joy, *Mt. Wilson Contr.* No. 142, *Astrophys. J.*, **46**, 1917 (313–339).

[5] Kapteyn, J. C., *Mt. Wilson Contr.* No. 147, p. 76, *Astrophys. J.*, **47**, 1918 (104–133, 146–178, 255–282), p. 266.

[6] Kapteyn, J. C., *Mt. Wilson Contr.* Nos. 82, 147, *Astrophys. J.*, **40**, 1914 (43–126), **46**, 1918 (104–133, 146–178, 255–282).

# THE MOTIONS IN SPACE OF SOME STARS OF HIGH RADIAL VELOCITY

By W. S. Adams and A. H. Joy

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Read before the Academy, April 28, 1919

The motion in space of the stars of very high radial velocity is of exceptional interest because such stars are usually of comparatively low intrinsic brightness and probably of small mass as well. It might, therefore, be expected that these stars would show the effects of stream motion strongly and that the components of their velocities might be related to the fundamental plane of the stellar galaxy.

As a basis for the study we selected all stars with radial velocities exceeding 80 km. per second for which proper motions and parallaxes are available. This gave a total of 37 stars with velocities ranging between 81 and 339 km., the latter the highest stellar radial velocity so far known. It is of interest to note that the largest velocities of approach and of recession are very nearly equal. The parallaxes of all but five of these stars have been derived by the spectroscopic method, the trigonometric values being used for the remainder.

After elimination of the solar motion the velocity-components and the apices of the stars relative to the centroid of the stellar system were determined by the aid of their proper motions, parallaxes and radial velocities. These components were in turn reduced to the plane of the galaxy and the galactic coördinates of the apices and the total velocities were calculated for all the stars.

Some of the results of the investigation are shown graphically in the accompanying figure. The plane is that of the galaxy with the apices shown in projection. Vectors drawn to the origin represent the direction and amount of motion in this plane.

A few of the more important conclusions may be indicated briefly.

1. The highest velocity in space found for any star is 494 km. for the ninth magnitude star A. G. Berlin 1366. Several other stars with larger parallaxes show values of nearly this amount. In all cases the components of motion in the plane of the galaxy exceed greatly those at right angles to it.

2. For the stars as a whole we find the two components in the galactic plane very nearly equal and more than two and one-half times as great as the component perpendicular to it. We find a similar result from a consideration of the latitudes of the apices. Only 6 out of the 37 stars have apices with latitudes exceeding 30°. It is clear, therefore, that the influence of the galactic condensation is very marked upon the motion of these stars.

3. Nearly an entire hemisphere in longitude is devoid of apices, the values all lying between 131° and 322°.

4. The velocity of the centroid of these stars is remarkably high. Thus even if we omit total velocities greater than 300 km. we find a value for the centroid of 74 km. This motion is almost wholly in the galactic plane.

5. The effect of stream motion among these stars is very marked. The axes for which the sum of the squares of the projected velocities is a maximum and a minimum have longitudes of 141° and 61°, respectively, and latitudes of +9° and −49°. The projections of these axes upon the galactic plane are

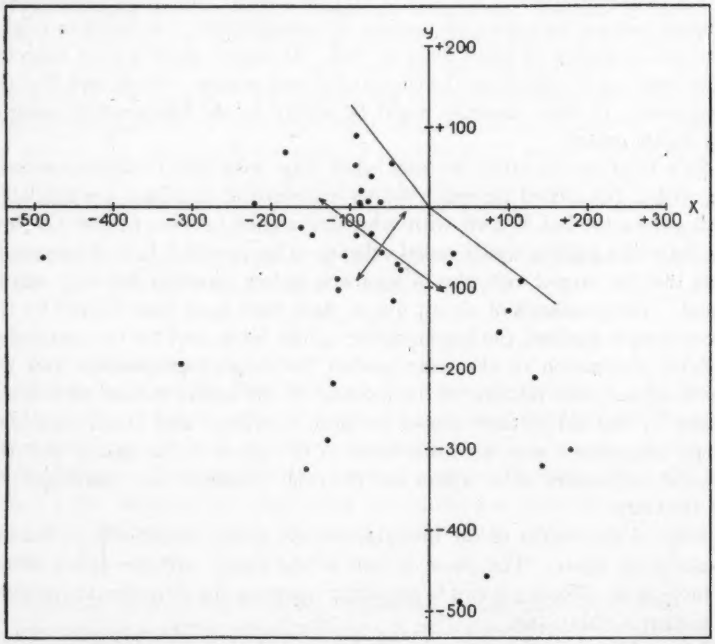


FIG. 1. PROTECTION ON GALACTIC PLANE OF APICES OF MOTION OF 37 STARS OF HIGH RADIAL VELOCITY

The vectors drawn from the origin represent the projected velocities in km per sec. The axes of the ellipse of intersection of the velocity-ellipsoid with the galactic plane are indicated by the two arrow-headed lines.

indicated in the figure. The mean square dispersion along the major axis is over twice that along the minor axis. The value for the major axis is in close agreement with that found by Strömberg from a discussion of the radial velocities of 260 dwarf stars, and with that of Raymond from 559 stars of large proper motion. All of these investigations indicate that the galactic longitude of the principal vertex for the stars of high velocity is considerably less than that for stars in general.

6. There appears to be some tendency for the stars to move along a line of galactic longitude about 260°, allowance being made for the motion of the centroid. This direction coincides nearly with that of the greatest star density as determined by various observers.

7. The stars of highest velocity, over 300 km., also appear to move along a line parallel to that of the major axis when the motion is referred to their common center. This is shown in the figure by the positions of the apices.

8. The average galactic latitude of the apices of the stars of high luminosity is nearly twice as great as that of the fainter stars, the values being 26° and 14°. The latitudes of the apices of only four of the fainter stars exceed 26°.

9. The average space-velocity of the stars of low luminosity is much larger than that of the brighter stars. Twenty-eight stars of average absolute magnitude 5.9 show a velocity of 216 km. Nine stars of absolute magnitude 0.4 a velocity of 130 km.

10. An extraordinarily large proportion of the stars in this list, 26 out of 37, are of types F and G. The successive types F, G, K and M show average space-velocities of 307, 156, 122 and 121 km. respectively, the weight of the determination for the last two types being rather low. Among the stars of type F those of earlier spectral type show the larger velocities. Thus the six stars with spectra between $F_0$ and $F_5$ inclusive have an average velocity of 365 km. as against 307 km. for all stars of the F type.

The most important result of this brief investigation is the evidence for the marked influence of the condensation of matter in the plane of the stellar galaxy upon the motions of these relatively faint stars. Their susceptibility to stream motion is perhaps another result of the same general influence which is, no doubt, gravitational in character. Probably the most peculiar fact in connection with these stars is the spectral type of the stars of highest velocity. That a type which we are accustomed to consider as intermediate in the scale of stellar development should contain so large a proportion of the most rapidly moving stars is difficult of explanation unless we may assume that these stars are of exceptionally small mass. Since the relationship of velocity to absolute magnitude seems to be fairly well established this hypothesis may be worthy of consideration.

# DEVIATIONS OF THE SUN'S GENERAL MAGNETIC FIELD FROM THAT OF A UNIFORMLY MAGNETIZED SPHERE

By F. H. Seares, A. van Maanen, and F. Ellerman
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Communicated by G. E. Hale. Read before the Academy, April 28, 1919

A communication to the Academy in 1917[1] describes the method used for the location of the sun's magnetic axis and gives values for the inclination, the period of revolution about the solar axis of rotation, and the epoch when the magnetic pole was on the central meridian. These results were based upon observations made on 63 days between June 8 and September 25, 1914. In the meantime the spectrograms of 11 additional days within the limits of this interval have been measured and reduced, and the elements defining the position of the axis have been revised with the following results (appended quantities are probable errors):

$$i = 6.^{\circ}0 \pm 0.^{\circ}4, \quad P = 31.52 \pm 0.28 \text{ days}$$
$$t_0 = 1914, \text{ June } 25.38 \pm 0.42 \text{ days, G. M. T.}$$
$$k = 0.99 \text{ (a constant inversely proportional to the polar field-strength)}$$

The modifications produced by the revision are unimportant, and, so far as the 1914 series of observations is concerned, these values may be accepted as final.

The uncertainty in the period is naturally large, for it has been derived from data covering less than four complete revolutions of the magnetic axis. Moreover, this limitation prevents any conclusion as to the constancy of the period. An improvement of the results in these particulars therefore requires additional observations, which should be distributed over a long interval. A beginning in this direction was made with a short series in September, 1916, which should reduce the uncertainty in the period well below a tenth of a day and make it possible to carry the longitude of the pole forward, without ambiguity as to the number of revolutions, to the coming sun-spot minimum, when further observations can be undertaken wihout risk of interference from the magnetic fields of spots.

The 1916 observations are now under discussion, and the indications are that the longitudes to be derived from them will agree closely with those calculated from the 1914 results. The period given above is therefore probably near the true value.

All discussions thus far described rest upon the assumption that the sun's general field is that of a uniformly magnetized sphere. The example of the earth, however, suggests that this hypothesis may be justified only to some rough degree of approximation, and that the field may not possess the uniformity hitherto presupposed.

We have therefore rediscussed the data from this standpoint, with results which seem to be conclusive. The investigation has had to meet the difficulty that the percentage errors in the data, in the nature of the case, are large; but there seems no doubt that there are irregularities in the field-strength at least, which cannot be accounted for on the basis of systematic or accidental error.

In order that the numerical quantities involved in the above solution—which we may refer to as the uniform-field solution—might be utilized as completely as possible, we have based the discussion upon the formulae there used, namely,

$$\left.\begin{array}{c} Ax + By = \Delta \\ x = k^{-1} \cos i, \quad y = k^{-1} \sin i \cos \lambda \\ Y = y/x = \tan i \cos \lambda \end{array}\right\} \tag{1}$$

$A$ and $B$ are known quantities, functions of the heliographic latitudes of the sun's center and the points observed; $\Delta$ is the observed displacement of the spectral line; and $i$ and $\lambda$ are the inclination of the magnetic axis and the longitude of the magnetic pole. $Y$ is a function of $i$ and $\lambda$, whose value is calculated for each day from all the measured $\Delta$'s for that day. The discussion of the resulting series of $Y$'s then leads to the required values of $i$, $t_0$, $P$, and finally to $k$, which is connected with $H_p$, the field-strength at the ple, by the relation

$$k = \frac{4}{CH_p}, \tag{2}$$

in which $C$ is a constant depending on the spectral line employed.

For the uniform-field solution, the values of $Y$ were calculated separately for three different lines. For the investigation of the deviations from the uniform field, the measures of the three lines were combined (this is possible since the values of $x$ for the three lines are sensibly equal) and the entire collection of data then subdivided according to zones of heliographic latitude as follows:

$$\begin{array}{ll} \text{I} & \varphi > +10° \\ \text{II} & +10° \gtrless \varphi \gtrless -10° \\ \text{III} & \varphi < -10° \end{array}$$

The outer limites for forces I and III vary somewhat, but are approximately 45°.

To the data within these limits we have applied equations (1), much as described above. In other words, we have derived separate uniform-field solutions for each of the zones. Had the three series of results agreed within the uncertainties affecting their determination, we could only have concluded that there is no evidence for the existence of appreciable deviations from the uniform field originally presupposed. As a matter of fact, we have found a large difference for $k$, which leads to the conclusion expressed above.

Certain details require comment. The fundamental equation—the first of (1)—cannot be applied directly to either zones I or III, because of the numeri-

cal values of the coefficients $A$ and $B$. There is no change in algebraic sign and the range in their values is so small that the simultaneous determination of $x$ and $y$ becomes practically indeterminate. For the middle zone the conditions are more favorable, but even here it has not seemed advisable to attempt a direct solution. Since the quantity $x$, as shown by the uniform-field solution, is apparently constant throughout the entire interval, we have preferred to solve by successive approximations, beginning with an assumed value of $x$.

We therefore write the fundamental equation in the form

$$B \tan i \cos \lambda = \Delta x' - A, \tag{3}$$

where for convenience $x' = k/\cos i$ has been substituted for $1/x$. The value of $x'$ from the uniform-field solution is 1.00, and this we use for the first approximation in deriving $Y = \tan i \cos \lambda$ from (3). We have for each zone, from all the data for each day, the normal equation

$$[BB] \tan i \cos \lambda = [B\Delta]\, x' - [AB] \tag{4}$$

The individual values of $Y$ are presumably in error, because of the assumed value for $x'$. It is easily shown, however, that neither the phase nor the period of the curve $Y = \tan i \cos \lambda$ is thereby affected. The entire effect goes into the amplitude. Using the first approximation for $x'$, we can therefore determine $t_0$ and $P$ for each zone, free from error, exactly as in the case of the uniform-field solution. The curves for $Y$ show that the results in each case are sensibly the same as those previously found, and we therefore adopt for each zone the values given above.

Since the weight of $\tan i \cos \lambda$ given by (4) is $[BB]$ the normal equation for $\tan i$ is

$$\tan i \, \Sigma\, [BB] \cos^2 \lambda = x' \, \Sigma\, [B\Delta] \cos \lambda - \Sigma\, [AB] \cos \lambda \tag{5}$$

where the outer summation symbol covers all the separate days. With this equation we find for the three values of $i$

$$\text{I} \quad 4^\circ.9 \pm 0^\circ.7; \qquad \text{II} \quad 6^\circ.7 \pm 0^\circ.5; \qquad \text{III} \quad 3^\circ.7 \pm 1^\circ.0 \tag{6}$$

With the aid of these values we now determine from the measures of each day a new approximation for $x'$, using the normal equation

$$[\Delta\Delta]\, x' = [A\Delta] + [B\Delta] \tan i \cos \lambda \tag{7}$$

which is at once derived from the fundamental equation. Noting that the weight of each $x'$ is $[\Delta\Delta]$ and combining the different days, we have for the mean $x'$

$$x' \Sigma[\Delta\Delta] = \Sigma[A\Delta] + \tan i \, \Sigma[B\Delta] \cos \lambda \tag{8}$$

The results of the second aproximation for $x'$ are

$$\text{I} \quad 0.97 \pm 0.009; \qquad \text{II} \quad 0.60 \pm 0.018; \qquad \text{III} \quad 0.96 \pm 0.008 \tag{9}$$

It thus appears that for zones I and III the original assumption was nearly correct; for the second zone, however, the change in $x'$ is large.

For the further approximations we have from (5) and (8), respectively,

$$d(\tan i) = \alpha dx' \tag{10}$$
$$dx' = \beta d(\tan i) \tag{11}$$

where

$$\alpha = \frac{\Sigma [B\Delta] \cos \lambda}{\Sigma [BB] \cos^2 \lambda} \qquad \beta = \frac{\Sigma [B\Delta] \cos \lambda}{\Sigma [\Delta\Delta]} \tag{12}$$

For zones I and III, $\alpha$ and $\beta$ are small quantities of the order of 0.01 or 0.02, and since $dx'$, the differences between the first and second approximations for $x'$, are only $-0.03$ and $-0.04$, respectively, equations (10) and (11) show that the corresponding values of $i$ and $x'$ given in (6) and (9) are final.

For zone II, $\alpha = 0.120$ and $\beta = 1.42$. We have found already, in order, $x'_1 = 1.00$, $\tan i_1 = 0.118$, $x'_2 = 0.60$, whence $dx' = -0.40$; and by successive substitution into (10) and (11) we derive $\tan i_2 = 0.070$, $x'_3 = 0.53$, $\tan i_3 = 0.062$, $x'_4 = 0.52$, $\tan i_4 = 0.061$. As no further change is produced by additional substitutions, $x'_4$ and $\tan i_4$ are adopted as the final values.

Calculating $k$ from $x'$ and collecting results, we have

| | | |
|---|---|---|
| For $\varphi = 45°$ N to $10°$ N, | $i = 4\overset{\circ}{.}9$, | $k = 0.96$ |
| $= 10$ N to 10 S, | $= 3.5$, | $= 0.52$ |
| $= 10$ S to 45 S, | $= 3.7$, | $= 0.96$ |

$$P = 31.52 \pm 0.28 \text{ days.}$$
$$t_0 = 1914, \text{ June } 25.38 \pm 0.42 \text{ days, G. M. T.}$$

The uncertainty in the inclinations averages about $0\overset{\circ}{.}7$, while that in the $k$'s is one or two units of the second decimal. Since the inclinations, periods, and epochs for the three zones are sensibly the same, the deviations from a uniform spherical field within the region from 45° N. to 45° S. are apparently restricted to the polar field-strength, which is inversely proportional to $k$. Here the variation with latitude is unexpectedly large—far greater than the uncertainty of the calculation. Unless the observations are affected by a large systematic error, which is itself a rather complicated function of the magnitude of the observed displacement $\Delta$, the change must be real.

The measures have all been made with the parallel-plate micrometer and it is difficult to believe that errors of the kind required to make the result illusory can have entered. Our experience would lead us to believe that important variations in the field-strength of the character indicated actually exist.

It is unexpected to find the $i$'s for all the zones smaller than that from the uniform-field solution. It is easily seen, however, that the two series of results stand in the proper relation to each other. In the uniform-field solution the displacements within the limits of zones I and III contribute about fifteen

times as much to the weight of $k$ as do those in zone II. The mean result for $k$ should therefore be sensibly the same as the values for zones I and III when treated separately, as proves to be the case ($k$ = 0.99, 96, and 96, respectively). The weight of the inclination, on the other hand, is derived largely from the observations of the equatorial zone. Moreover, $i$ is calculated after $k$ has been found, so that the conditions in the uniform-field solution are analogous to those of the first approximation of zone II. In this approximation the equatorial observations combined with $x' = 1.00$—the equivalent of $k$ = 0.99—gave $i = 6°.7$, which is nearly the same as the value found from all the data. The measures of zones I and III combined with $k$ = 0.99, as we know from the zonal analysis, must lead to relatively small values for $i$, amounting to about 4°. The mean therefore lies between these two extremes, with a preponderance of weight in zone II. This accounts for the value of 6°.0 originally found—a mean result which is in excess of all the inclinations found by treating the zones separately. It is only when the equatorial observations are discussed by themselves that the true value of $k$ for this region reveals itself or has any appreciable influence upon the solution; but when once found the inclination is necessarily decreased.

We are under great obligation to Miss Wolfe of the Computing Division who has rendered much assistance with the extensive numerical calculations required for the discussion of the data.

[1] Seares, F. H., van Maanen, A., and Ellerman, F., these PROCEEDINGS **4**, 1918, (4–9).

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## *ON THE PROBLEM OF THE PRODUCTION OF FAT FROM PROTEIN IN THE DOG*

BY H. V. ATKINSON AND GRAHAM LUSK

CORNELL UNIVERSITY MEDICAL COLLEGE, NEW YORK CITY

Read before the Academy, April 28, 1919

In eight experiments in a series of thirteen, after giving meat in large quantities (700 to 1300 grams) to a dog weighing 11 kgm., the respiratory quotients during the height of protein metabolism were between 0.793 and 0.800. A calculation showed that under these conditions the retained carbon residue of the protein metabolized was held back in such a form that, had it been oxidized, the respiratory quotient of this retained pabulum would have been 0.85. This would represent the oxidation of material half of whose calories were composed of fat and half of carbohydrate. The dog showed quotients of 0.82 and above only after the larger quantities of meat were given (1000 grams or more). It was extremely difficult to induce the dog to take meat in sufficient quantity to indicate a considerable production of fat from protein. Incidentally it was observed that the basal metabolism of a dog fed with meat in large quantity for a time and then caused to revert to a 'standard diet'

(meat, 100 grams; lard, 20 grams; and biscuit meal, 100 grams) remained persistently (even after 2½ weeks) at a higher level than had obtained prior to the meat ingestion. This confirms F. G. Benedict's idea of a higher basal metabolism in the presence of 'surplus' cellular nitrogen, or the 'improvement quota' of protein according to Rubner's suggestive terminology. When meat was given after the partial depletion of the body cells of their 'improvement quota', protein was retained in greater measure, less protein was metabolized and the heat production was therefore lower than on a subsequent day (experiments 46, 47; 51, 54).

The following table gives the method of calculation followed:

*Experiment 55. N in urine per hour = 1.58 grams*

| | grams | | grams | | |
|---|---|---|---|---|---|
| N-$CO_2$ | 14.77 | N-$O_2$ | 13.35 | N-cals | 41.89 |
| Resp. $CO_2$ | 11.42 | Resp. $O_2$ | 10.42 | Deposit cals | 9.92 |
| Dif. | 3.35 | | 2.93 | Indirect cals | 31.97 |
| | | | | Direct cals | 31.98 |

R. Q. of deposit = 0.83

R. Q. per hour—0.81, 0.74, 0.80, 0.84. R. Q. for whole period = 0.797

In the above table:

N-$CO_2$ is the amount of $CO_2$ derivable from the protein metabolism during an hour.

N-$O_2$ is the amount of $O_2$ necessary to oxidize the protein metabolized in one hour.

Resp. $CO_2$ and Resp. $O_2$ are the amounts of $CO_2$ and $O_2$ which were actually respired during an hour.

The difference represents (1) the $CO_2$ which would have been expired had all the retained carbon of the protein metabolism been oxidized and (2) the $O_2$ which would have been employed in that process. The relation between the volumes of these two gases indicates that the material retained and unoxidized would have yielded a respiratory quotient of 0.83, which indicates the retention of a pabulum, approximately half of the calories of which were derived from fat and half from glucose.

N-cals. is the quantity of heat which would have been eliminated by the dog had all the protein metabolized by the dog been completely oxidized. From this is subtracted the number of calories estimated to have been retained as a mixture of fat and glucose aforesaid. The difference represents the calories as calculated by *indirect calorimetry*, which in this case agrees exactly with those directly measured by the calorimeter by the method of *direct calorimetry*.

The following summary of results is appended.

*Effect of meat ingestion on hourly metabolism*

| EXP. NO. | DATE | FOOD | NUMBER OF HOURS | URINE N | R. Q. | CALORIES | | C DEPOSITED | R. Q. OF DEPOSIT | HOURS AFTER FOOD |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Indirect | Direct | | | |
| | 1919 | | | | | | | | | |
| 27 | Feb. 6 | Basal | 2 | 0.15 | 0.84 | 15.92 | 16.08 | | | (Weight = 11.24 kgm.) |
| 29 | Feb. 17 | Meat, 700 g. | 3 | 1.40 | 0.820 | 29.97 | 30.01 | 0.60 | 0.73 | 4, 5, 6 |
| 30 | Feb. 18 | Meat, 800 g. | 3 | 1.46 | 0.800 | 31.47 | 32.75 | 0.66 | 0.83 | 4, 5, 6 |
| 31 | Feb. 19 | Meat, 900 g. | 3 | 1.47 | 0.787 | 34.33 | | 0.48 | 0.96 | 4, 5, 6 |
| 32 | Feb. 20 | Meat, 1000 g | 2 | 1.46 | 0.797 | 34.27 | 34.50 | 0.42 | 0.87 | 4, 5 |
| | | | 2 | 1.46 | 0 820 | 35.50 | 37.20 | 0.23 | 0.62 | 6, 7 |
| 33 | Feb. 21 | Meat, 1100 g. | 2 | 1.45 | 0.831 | 31.65 | 31.36 | 0.56 | 0.68 | 4, 5 |
| | | | 2 | 1.45 | 0.843 | 35.28 | 34.54 | 0.25 | 0.49 | 6, 7 |
| 34 | Feb. 24 | Meat, 1080 g.* | 2 | 1.57 | 0.800 | 34.00 | 34.12 | 0.70 | 0.83 | 4, 5 |
| 35 | Feb. 26 | Basal | 3 | 0.27 | 0.82 | 19.74 | 19.59 | | | (Weight = 12.07 kgm.) |
| 36 | Feb. 27 | Basal | 3 | 0.20 | 0.83 | 18.25 | 17.16 | | | |
| 37 | Feb. 28 | Basal | 2 | 0.17 | 0.85 | 17.30 | 16.95 | | | |
| 38 | Mar. 1 | Basal | 2 | 0.15 | 0.82 | 18.21 | | | | |
| 39 | Mar. 3 | Basal | 3 | 0.15 | 0.85 | 17.57 | 17.22 | | | |
| 43 | Mar. 12 | Basal | 2 | 0.15 | 0.81 | 17.08 | 16.99 | 0 | | (Weight = 11.50 kgm.) |
| 46 | Mar. 17 | Meat, 1200 g. | 3 | 1.02 | 0.796 | 26.57 | 28.10 | 0 | | 5, 6, 7 after 1 day's fast |
| 47 | Mar. 18 | Meat, 800 g. | 3 | 1.44 | 0.795 | 29.90 | 30.77 | 0.77 | 0.84 | 5, 6, 7 |
| 48 | Mar. 19 | Meat, 800 g. | 4 | 1.35 | 0.793 | 29.37 | 30.27 | 0.61 | 0.86 | 5 to 8 |
| 49 | Mar. 22 | Basal | 2 | 0.23 | 0.79 | 17.72 | 17.54 | | | |
| 50 | Mar. 24 | Basal | 2 | 0.16 | 0.84 | 17.26 | 16.87 | | | |
| 51 | Mar. 28 | Meat, 800 g. | 4 | 1.02 | 0.795 | 27.04 | 27.52 | | | 5 to 8 after 4 days' fast |
| 54 | Apr. 15 | Meat, 800 g. | 4 | 1.41 | 0.794 | 31.07 | 30.57 | 0.59 | 0.86 | 5 to 8 |
| 55 | Apr. 16 | Meat, 1000 g. | 4 | 1.58 | 0.797 | 31.97 | 31.98 | 0.91 | 0.83 | 5 to 8 |
| 56 | Apr. 19 | Meat, 1300 g. | 4 | 1.47 | 0.826 | 31.62 | 33.25 | 0.59 | 0.71 | 5 to 8 after 1 day's fast |

* Standard diet at 5 p.m. and thereafter daily until March 15.

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## *ON THE TWIST IN CONFORMED MAPPING*

By T. H. Gronwall

Range Firing Section, Aberdeen Proving Ground

Communicated by E. H. Moore, April 29, 1919

Note II on Conformal Mapping under aid of Grant No. 207 from the Bache Fund.

Let $w = w(z)$ be a power series in $z$, convergent for $|z|<1$ and such that the circle $|z|<1$ is mapped conformally on a *simple* (that is, simply connected and nowhere overlapping) region in the $w$-plane. By a linear transformation $w^1 = aw + b$, we may reduce $w(z)$ to the form $z + a_2z^2 + \ldots + a_nz^n +$

. . . . For the comparison of maps of different regions, two geometrical concepts are of fundamental importance: the *distortion* = $|dw/dz|$, or the ratio of the lengths of corresponding line elements in the $w$- and $z$-planes, and the *twist* = imaginary part of log $(dw/dz)$, or the angle between corresponding line elements, this angle being always taken between $-\pi$(excl.) and $\pi$ (incl.). Koebe[1] has shown that on the circle $|z| = r$, $0<r<1$, both $|dw/dz|$ and $|z|$ lie between positive bounds which depend on $r$ alone, and the writer[2] has determined the exact values of these bounds.

It is the purpose of the present note to state the corresponding result in regard to the twist:

*When the analytic function*

$$w = z + a_2z^2 + \ldots + a_nz^n + \ldots$$

*maps the circle* $|z|<1$ *on the interior of a simple region* $D$ *in the* $w$*-plane, the twist* $\tau$ *satisfies the following inequalities for* $|z| = r$ *and* $0<r<2^{-\frac{1}{2}}$

$$-4 \arcsin r<\tau< 4 \arcsin r, \tag{1}$$

*except when*

$$w = \frac{z(1 - \cos\beta \cdot e^{\alpha i}z)}{(1 - e^{(\alpha+\beta)i}z)^2}, \tag{2}$$

*where* $\alpha$ *and* $\beta$ *are real, and* $\cos\beta = r$. *In this case,* $\tau$ *attains the upper or lower bound in* (1) *for* $z = re^{-\alpha i}$ *according as* $\beta = +\arccos r$ *or* $\beta = -\arccos r$. *For* $2^{-\frac{1}{2}} \leqq r<1$, *we have only the inequality included in the definition of* $\tau$

$$-\pi<\tau\leqq\pi, \tag{3}$$

*and no single class of functions analogous to* (2) *reaching the upper and lower bounds can be assigned.*

*When the region* $D$ *is* CONVEX, *we have for* $|z| = r$ *in the whole interval* $0<r<1$

$$-2 \arcsin r<\tau< 2 \arcsin r \tag{4}$$

*except when*

$$w = \frac{z}{1 - e^{(\alpha+\beta)i}z}, \quad \cos\beta = r, \tag{5}$$

*the upper and lower bounds being then attained as above.*

The proof is similar to that of the distortion theorem outlined before.[2] The region $D$ is approximated by rectilinear polygons, for which we have the formula of Schwarz

$$\frac{dw}{dz} = (1 - e^{\alpha_1 i}z)^{\mu_1}(1 - e^{\alpha_2 i}z)^{\mu_2} \ldots (1 - e^{\alpha_m i}z)^{\mu_m} \tag{6}$$

whence, for $z = re^{\theta i}$,

$$\tau = \Sigma_1^m - \mu_\nu \arctan\frac{r\sin(\theta + \alpha_\nu)}{1 - r\cos(\theta + \alpha_\nu)} + 2k\pi i, \tag{7}$$

where the arctangents are all taken between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$, and the integer $k$ is chosen so as to make $-\pi<\tau\leqq\pi$. Since the polygon is not overlapping, we have

$$\Sigma\mu_\nu = -2, \quad -3\leqq\mu_\nu\leqq 1 \quad (\nu= 1,2, \ldots,m)$$

For a fixed value of $m$, we consider the set of all the values of $\alpha_1, \ldots, \alpha_m$, $\mu_1, \ldots, \mu_m$ for which (6) maps $|z|<1$ on a *simple* polygon, and it is first shown that this set is *closed*. On this set, and for $-\pi\leqq\theta\leqq\pi$ the expression (7) has therefore a maximum and a minimum, which are found by observing that

$$\arctan\frac{r\sin\varphi}{1-r\cos\varphi}$$

has a maximum = arcsin $r$ for $\varphi$ = arccos $r$ and a minimum = − arcsin $r$ for $\varphi$ = − arccos $r$. In the discussion of (7), it is necessary to distinguish the case where all $\mu$ are negative, corresponding to a convex polygon, from the general case where some $\mu$ are positive; for this reason, the convex regions appear separately in the statement of the theorem. By considerations of continuity, and the use of elementary properties of harmonic functions, it is finally shown that the upper and lower bounds in (1) and (4) are reached in the cases (2) and (5) only, and that no closer bounds than (3) can be found for $2^{-\frac{1}{2}}\leqq r<1$ in the non-convex case.

Regarding (2) and (5), we observe that by rotating the $z$- and $w$-planes through the angle $-\alpha$ about their origins, we may make $\alpha = 0$; in this case, the circle $|z|<1$ is mapped by (2) on the $w$-plane slit along the straight line segment

$$w = \frac{i}{4\sin\beta} - ie^{-2\beta i}t, \qquad (0\leqq t\leqq + \quad),$$

and by (5) on the half plane in which the real part of $we^{\beta i}$ is greater than $-\frac{1}{2}$.

[1] Koebe, *Göttingen, Nachr. Ges. Wiss.*, 1909, (73).
[2] Gronwall, *Paris, C. R. Acad. Sci.*, 162, 1916, (249).

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## *SEVENTEEN SKELETONS OF MOROPUS; PROBABLE HABITS OF THIS ANIMAL*

By Henry Fairfield Osborn
American Museum of Natural History, New York City

Read before the Academy, April 29, 1919

*Moropus* is an aberrant perissodactyl, closely related to the family of the Titanotheres and more remotely to that of the Horses. It occurs in the Lower Miocene age in France and North America, and its ancestors have been

traced back to the Upper Eocene in both countries; it is thus of Holarctic distribution, and while very rare, it must have been perfectly adapted to its environment, because it survived the majority of perissodactyls and occurs in the Pliocene of Europe and England and will not improbably be found in the North American Pliocene.

The habits and habitat of the animal have always presented a very difficult problem. The skeleton presents the most noteworthy exception to Cuvier's law of correlation. All the foot bones which were discovered since Cuvier's

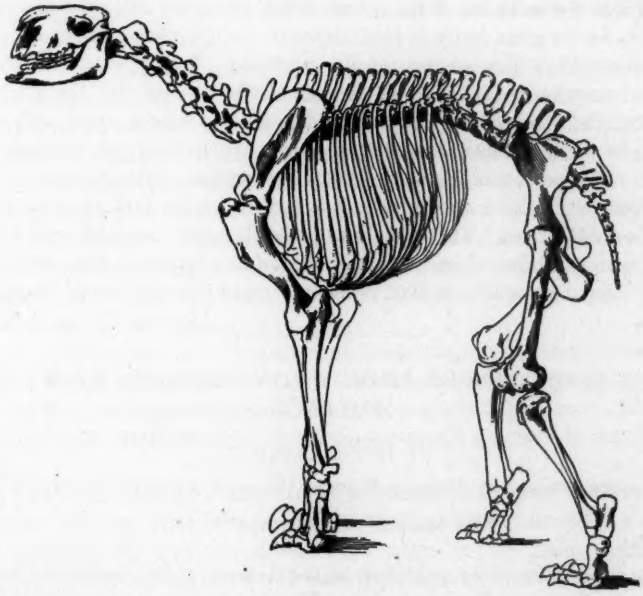

FIG. 1

Mounted skeleton of *Moropus cookei* in The American Museum of Natural History. One of the seventeen. Drawing one twenty-sixth natural size.

time consisted of large deeply-cleft terminal phalanges and were grouped with the edentates, especially the plantigrade sloths. All the teeth which were discovered, on the other hand, were grouped with the perissodactyl ungulates. It was not until H. Filhol discovered the nearly complete skeleton of *Macrotherium* that he was enabled to prove that the chalicotheres were of composite adaptive structure, with the teeth of perissodactyls and the claws of edentates. *Macrotherium* is very similar to the American *Moropus*.

Great light was thrown upon the structure of *Moropus* through the explorations of the Carnegie Museum by Holland and Peterson, described in 1914, from materials collected in the famous Agate Spring Quarry, Sioux County,

Nebraska, discovered by James H. Cook in 1897. After the lapse of the Carnegie researches and explorations, the American Museum entered this quarry and through five years of continuous exploration (1911–1916) an irregular area within a square of about 36 feet yielded nearly complete skulls of ten individuals and skeletal parts of seventeen individuals all together. From this wonderful material it has been possible to supplement the descriptions of Holland and Peterson and to present for the first time the proportions and pose, by which we may estimate the habits of this animal. We reach the conclusion that the Moropus type was not plains living, but forest living; that it was the seclusion of the forests which protected this type and which accounts for its great rarity in fossil deposits, for it is characteristic of forest-living forms that they are not readily entombed. We form an entirely different conception of the habits of the animal when we observe the extremely long fore limbs, which are not of the digging or fossorial type, and which thus belie the apparently fossorial or digging structure of the terminal phalanges. It appears more probable that these terminal claws were used partly for purposes of offense and defense, but largely for the pulling down of the branches of the trees. The animal was probably forest living like the African okapi, with which in its general body and head proportions it has many analogies. Like the okapi it survived through retreat to the recesses of the forests.

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## *THE STRUCTURE OF ATOMS AND THE OCTET THEORY OF VALENCE*

By Irving Langmuir

Research Laboratory, General Electric Company, Schenectady, New York

Read before the Academy, April 29, 1919

In a paper soon to be published in the *Journal of the American Chemical Society*, I will give a new theory of the structure of atoms and molecules based upon chemical data. This theory, which assumes an atom of the Rutherford type, and is essentially an extension of Lewis' theory of the 'cubical atom,'[1] may be most concisely stated in terms of the following postulates.

1. The electrons in atoms are either stationary or rotate, revolve or oscillate about definite positions in the atom. The electrons in the most stable atoms, namely, those of the inert gases, have positions symmetrical with respect to a plane called the equatorial plane, passing through the nucleus at the center of the atom. No electrons lie in the equatorial plane. There is an axis of symmetry (polar axis) perpendicular to the equatorial plane through which four secondary planes of symmetry pass, forming angles of 45° with each other. These atoms thus have the symmetry of a tetragonal crystal.

2. The electrons in any given atom are distributed through a series of concentric (nearly) spherical shells, all of equal thickness. Thus the mean

radii of the shells form an arithmetic series 1, 2, 3, 4, and the effective areas are in the ratios $1:2^2:3^2:4^2$.

3. Each shell is divided into cellular spaces or cells occupying equal areas in their respective shells and distributed over the surface of the shells according to the symmetry required by postulate 1. The first shell thus contains 2 cells, the second 8, the third 18, and the fourth 32.

4. Each of the cells in the first shell can contain only one electron, but each other cell can contain either one or two. All the inner shells must have their full quotas of electrons before the outside shell can contain any. No cell in the outside layer can contain two electrons until all the other cells in this layer contain at least one.

5. When the number of electrons in the outside layer is small, these electrons arrange themselves over the underlying ones, being acted on by magnetic attractive forces. But as the charge on the kernel or the number of electrons in the outside layer increases, the electrostatic repulsion of the underlying electrons becomes predominant and the outer electrons then tend to rearrange themselves so as to be as far as possible from the underlying ones.

6. The most stable arrangement of electrons is that of the pair in the helium atom. A stable pair may also be held by: (*a*) a single nucleus; (*b*) two hydrogen nuclei; (*c*) a hydrogen nucleus and the kernel of another atom; (*d*) two atomic kernels (very rare).

7. The next most stable arrangement of electrons is the octet; that is, a group of eight electrons like that in the second shell of the neon atom. Any atom with atomic number less than twenty, and which has more than three electrons in its outside layer tends to take up enough electrons to complete its octet.

8. Two octets may hold one, two, or sometimes three pairs of electrons in common. One octet may share one, two, three or four pairs of its electrons with one, two, three or four other octets. One or more pairs of electrons in an octet may be shared by the corresponding number of hydrogen nuclei. No electron can be shared by more than two octets.

The inert gases are those having atoms in which all the cells in the outside shell have equal numbers of electrons. Thus according to the first four postulates the atomic numbers of the inert gases should be 2, 10, 18, 36, 54, and 86 in agreement with fact.

All atoms with an atomic number greater than that of helium, have as their first shell a pair of electrons close to the nucleus. The line connecting the two electrons establishes the polar axis for the atom. Neon has in its second shell eight electrons, four in each hemisphere (i.e., above and below the equatorial plane), arranged symmetrically about the polar axis. The eight electrons are thus nearly at the corners of a cube. In argon there are eight more electrons in the second shell.

The eight electrons in the third shell of the atom of iron are arranged over the underlying electrons in the second shell. The two extra electrons in the

atom of nickel are placed in the polar axis. Beyond nickel the electrons in the atom cannot be held by magnetic forces, and thus tend to rearrange themselves so as to be placed as far as possible from the underlying electrons. This leads to an explanation of the chemical and magnetic properties of copper, zinc, etc.

Krypton has in its third shell nine electrons in each hemisphere, symmetrically placed with respect to the polar axis and to the four electrons in the second shell. The ninth electron in each hemisphere goes into the polar axis. Xenon is like krypton, except that it has twice as many electrons in its third shell. Beyond Xenon eighteen electrons in the fourth shell can be held by magnetic forces over the eighteen cells of the third shell, so that lutecium, the eighteenth element beyond Xenon marks the last of the rare earth elements. The electrons in the outside shell of the atoms beyond this element are arranged as far as possible so as to leave eighteen empty spaces over the underlying electrons. In this way it is possible to explain the chemical and magnetic properties of tantallum and tungsten as contrasted to those of the rare earths.

Niton has sixteen electrons in each hemisphere of its fourth shell. These are placed symmetrically with respect to the polar axis and the eight underlying electrons.

This theory of atomic structure explains in a satisfactory way most of the periodic properties of all the elements including those of the eighth group and the rare earths. It lends itself especially well to the explanation of the so-called physical properties, such as boiling-points, freezing-points, electric conductivity, etc. For the details of its application to specific elements the paper in the *Journal of the American Chemical Society* should be consulted.

Postulates 6, 7 and 8 lead directly to a new theory of valence which we may call the Octet Theory. This theory may be stated in terms of the equation

$$e = 8n - 2p \qquad (1)$$

where $e$ is the total number of available electrons in the shells of all the atoms in a molecule; $n$ is the number of octets forming the outside shells of the atoms and $p$ is the number of pairs of electrons held in common by the octets (Postulate 8). If we let $E$ be the number of electrons in the 'shell' of an atom then $e = \Sigma (E)$. The value of $E$ for a given atom, at least in case of the first twenty elements, corresponds to the ordinal number of its group in the periodic table. Thus we have the following values of $E$:—one for hydrogen, lithium, sodium, etc., two for magnesium, three for boron, aluminum, etc.; four for carbon and silicon, five for nitrogen and phosphorus; six for oxygen and sulphur; seven for the halogens, and zero for the inert gases.

The above equation expresses the fact that every pair of electrons held in common between two octets results in a decrease in two in the total number of electrons needed to form the shells of the atoms in the molecules. It also implicitly expresses the fact that *all the electrons* in the shells of the atoms forming a molecule form part of one or two of the octets in the molecule.

It seems that this simple equation is a practically complete statement of a theory of valence that applies with very few exceptions to all compounds formed from the first twenty elements. With some modifications it applies also to most compounds of other elements. In the case of organic compounds it is found that each pair of electrons held in common between atoms corresponds exactly to the valence bond used in the ordinary theory of valence. It is therefore proposed to define valence as the number of pairs of electrons which a given atom shares with others. In view of the fact known that valence is very often used to express something quite different, it is recommended that the word *covalence* be used to denote valence defined as above.

Equation (1) expresses the fact that the number of covalence bonds in a molecule must be related to the number of available electrons in the molecule. A simple mathematical analysis[2] shows that all structural formulas written according to the ordinary valence theory in which the valence for each element is taken equal to 8–E, will satisfy Equation (1). Thus the Octet Theory requires no modification in any formula written with the following valencies; carbon-four, nitrogen-three; oxygen-two; chlorine-one and hydrogen-one. In some cases, however, the Octet Theory suggests that other formulas besides those usually adopted may be possible. Whenever the old theory of valence has assumed valencies other than those mentioned above, such as five for nitrogen or phosphorus; four or six for sulphur; three, five or seven for chlorine, etc., the Octet Theory gives quite different structural formulas from those usually assumed. This is readily seen when it is considered that the covalency of an element according to the Octet Theory can never exceed four, since there are only four pairs of electrons in an octet.

A careful examination of the cases showing a discrepancy between the old and new theories furnishes the strongest kind of evidence in support of the Octet Theory. The non-existence of such compounds as $H_4S$, $H_6S$, $SCl_6$, $NCl_5$, $NH_5$, etc., is in full accord with the theory as is also the existence of $SO_2$, $SO_3$, $N_2O_5$, $HNO_3$, $NH_4Cl$, etc. In these latter cases, however, the formulas written are different from those usually adopted. For example, the covalence of sulphur is three in $SO_2$, four in $SO_3$; that of nitrogen is four in $N_2O_5$, $HNO_3$, and $NH_4Cl$. These covalencies are, however, not *assumed* as in the ordinary valence theory, but are *derived* from Equation (1), which is the *same* equation as that which applies to all ordinary organic compounds. In a similar way it is found that the Octet Theory fully explains the structures of such compounds as $N_2O$, $N_2O_3$, $N_2O_4$, $HN_3$, $N(CH_3)_4Cl$, $H_3PO_3$, $H_3PO_4$, $HClO$, $HClO_2$, $HClO_3$, $HClO_4$, $H_2O_2$, and even so-called complex compounds such as $B(CH_3)_3$, $NH_3$, $K_2PtCl_2.2NH_3$, $KBF_4$, $Na_2S_5$, etc.

From this viewpoint a very large number of compounds previously considered by Werner are found to be typical primary valence compounds not essentially different in their structures from organic compounds. It is especially significant that the structure of these compounds is found from Equation (1), without any additional assumptions. Thus we are lead to a *single theory* of

valence which explains and coördinates the separate valence theories that we have needed in the past.

There are many facts not previously well understood which are very readily explained by the new theory. For example, the fact that we have weak and strong acids, weak and strong bases, but no 'weak' and 'strong' salts, is automatically explained.

The theory indicates that all salts consist of negative and positive ions even when in the solid condition, and that no pair of electrons are held in common between the negative and positive groups. Thus in sodium chloride the covalence of both sodium and chlorine is zero, and this fact explains the nonexistence of molecules of sodium chloride shown by the X-ray crystal analysis. When sodium chloride is dissolved in water the water does not cause ionization, but simply causes the separation of atoms already ionized. This direct result of the Octet Theory is in full accord with experiment and with Milner's recent theory of strong electrolytes. *London, Phil. Mag.*, **35**, 1918, (214, 354).

In the field of organic compounds the theory fits the facts particularly well. Although in the case of compounds like $SF_6$, $H_2SiF_6$, etc., there is very definite evidence that the *kernels* of the atoms of sulphur and silicon are cubical in shape, there is the strongest evidence that in organic compounds the carbon atom has the eight electrons of its octet drawn together into four pairs, arranged at the corners of a tetrahedron. This is in full accord with the fact that in $SF_6$, and $H_2SiF_6$, the central atom has given up electrons to the surrounding atoms, so that the cubical kernels do not share any electrons with the other atoms, while in organic compounds the carbon atoms always share four pairs of electrons with adjacent atoms. From this we must conclude that a pair of electrons held in common by two octets acts as if it were located at a point between the two atoms. This conclusion, which can be reduced from the properties of a very few simple organic compounds is found to apply apparently without exception to compounds of nitrogen, sulphur, phosphorus, and even cobalt compounds, etc. It seems to explain all the cases of stereoisomerism that I am familiar with. For example, in the amine oxides, $NR_1R_2R_3O$, nitrogen is quadricovalent, so that these substances exist as optical isomers, just as in the case of a carbon atom attached to form different groups.

The isomerism of compounds of tervalent nitrogen such as ketoximes, hydrazones, ozazones, and diazo-compounds, etc., is readily accounted for, as well as the absence of isomers among tertrary amines, etc. Not only are the substituted ammonium compounds fully explained, but also the sulfonium, phosphonium, and oxonium compounds. Thus the structures of $S(CH_3)_3OH$, $S(CH_3)_4Cl_2$, $(C_2H_5)_2O.HCl$, etc., are readily found from the Octet Theory and their salt-like character is explained. The covalence of the central atom in the above compounds is three, four and three respectively.

When the values of $e$ and $n$ are both the same for two or more compounds it is evident according to the Octet Theory that these may have practically identical structures. An example of this kind is found in $N_2$ and CO. The total

number of electrons in each molecule (including those in their kernels) is fourteen. Evidence is given in the paper in the *Journal of the American Chemical Society* that the structures of these two molecules is identical, except for the fact that in one case there are two nuclei of seven positive charges each, while in the other there are nuclei of six and eight charges, respectively. These molecules are, however, exceptional, in that the molecule consists of a *single octet* arranged around a complex kernel.

Another example of a pair of compounds which according to the Octet Theory should have similar structures occurs in the case of $CO_2$ and $N_2O$. For each of these molecules $n = 3$, $e = 16$ and therefore $p = 4$. The best method of testing this conclusion lies in comparing the '*physical*' properties of the two substances. The 'chemical' properties depend primarily on the ease with which the molecules can be broken up, and thus is a measure of the internal forces within the molecule, which depend to a large extent on the charges on the kernels. The so-called 'physical' properties on the other hand depend on the stray field of force outside of the molecule, and this naturally depends rather on the arrangement of the outside electrons.

As a matter of fact we find that most of the physical properties of these two gases are practically identical.

The following data taken from Landolt-Börnstein tables and Abegg's hand book illustrate this.

| | $N_2O$ | $CO_2$ |
|---|---|---|
| Critical pressures | 75 | 77 atmos. |
| Critical temperatures | 35.4° | 31.9° |
| Viscosity at 20°C | $148 \times 10^{-6}$ | $148 \times 10^{-6}$ |
| Heat Conductivity at 100°C | 0.0506 | 0.0506 |
| Density of Liquid at −20° | 0.996 | 1.031 |
| Density of Liquid at +10° | 0.856 | 0.858 |
| Refractive index of liquid. D line, 16°C | 1.193 | 1.190 |
| Dielectric constant of liquid at 0° | 1.598 | 1.582 |
| Magnetic susceptibility of gas at 40 atmos. 16°C | $0.12 \times 10^{-6}$ | $0.12 \times 10^{-6}$ |
| Solubility in water 0° | 1.305 | 1.780 |
| Solubility in alcohol at 15° | 3.25 | 3.13 |

Both gases form hydrates $N_2O.6H_2O$ and $CO_2.6H_2O$. The vapor pressure of the hydrate of $N_2O$ is 5 atmospheres at −6°C. while the hydrate of $CO_2$ has this vapor pressure at −9°C. The heats of formation of the two hydrates are given respectively as 14,900 and 15,000 calories per gram molecule.

The surface tension of liquid $N_2O$ is 2.9 dynes per cm. at 12.°2, while $CO_2$ has this same surface tension at 9.0°.

Thus $N_2O$ at any given temperature has properties practically identical with those of $CO_2$ at a temperature 3° lower.

These results establish the similarity of outside structure of the molecules.

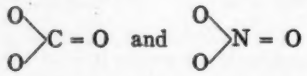
There is one property however, which is in marked contrast to those given above. The freezing-point of $N_2O$ is $-102°$ while that of $CO_2$ is $-56°$. This fact may be taken as an indication that the freezing-point is a property which is abnormally sensitive to even slight differences in structure. The evidences seem to indicate that the molecule of $CO_2$ is slightly more symmetrical, and has a slightly weaker external field of force than that of $N_2O$. Such differences could easily be produced by the difference in the charges on the kernels.

There are many other examples of compounds having similarly formed molecules. It will be convenient to call these *isosteric compounds* or *isosteres*. These may be defined as compounds whose molecules have the same number and arrangement of electrons.

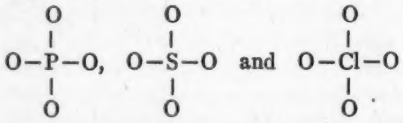
Another example of a pair of isosteres is that of $HN_3$ and HCNO. The similarity of properties should be most marked in the salts of these acids. The available data on solubilities and crystalline form seem to show that the salts of these two acids are very closely similar in physical properties.

This relationship of compounds may be carried much further. Thus, according to the Octet Theory, we should regard $CH_4$ as an isostere of the $NH_4$ ion. The electric charge on the ion prevents a direct comparison of physical properties. Other examples are:

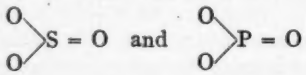
The carbonate and nitrate ions:



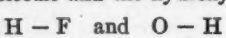
The orthophosphate, sulphate and perchlorate ions:



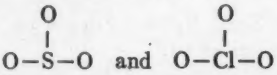
Sulfur trioxide and the metaphosphate ion:



The hydrofluoric acid molecule and the hydroxyl ion:



The sulfite and the chlorate ions:



We may attribute the differences in physical properties in all these cases to the effect of the differences in the electric charges.

Lewis has already pointed out that a theory of the kind outlined in this paper explains satisfactorily the facts which have led many chemists to assume polar valence. For example, the chlorine atom in chlor-acetic acid, because of the relatively large charge on its kernel, as compared for example with a carbon atom, tends to displace towards itself the electrons holding it to the carbon atom. This effect is transmitted with gradually decreasing intensity to the further end of the molecule, where it results in drawing the pair of electrons which holds the hydrogen nucleus to the octet of the oxygen atom, away from the hydrogen nucleus. Another way of looking at the effect is to consider that the positive kernel of the oxygen atom is displaced toward the hydrogen nucleus, and thus tends to weaken the force holding it. This effect makes it easier for the hydrogen nucleus to separate from the rest of the molecule as a positive ion. It is felt that this explanation can be applied in general to explain nearly all cases where polar valence bonds have been assumed in the past. This question will be discussed in more detail in the second paper to be published in the *Journal of the American Chemical Society*.

[1] Lewis, G. N., *J. Amer. Chem. Soc.*, 38, 1916, (762–785).

[2] This will be published in full in a paper soon to be submitted to the *Journal of the American Chemical Society*. This second paper will deal in some detail with the application of the Octet Theory to organic chemistry, particularly to nitrogen, sulphur, compounds, etc. The stereoisomerism of such compounds will be discussed.

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## *A NEW INSTRUMENT FOR MEASURING PRESSURES IN A GUN*

By A. G. Webster and L. T. E. Thompson

Ballistic Institute, Clark University, Worcester, Massachusetts*

Read before the Academy, April 29, 1919

It is now over fifty years since the crusher gauge was invented for measuring the maximum pressure developed in a gun. This apparatus has probably gone through fewer changes than almost any physical instrument except the telegraphic sounder. It is looked upon by all experts as inaccurate, and should be superseded. We have developed an apparatus which shows not only maximum pressure, but also the pressure at any time while the projectile is in the barrel; that is, it gives the curve which represents the pressure as a function of the time. Attention is called to the fact that this curve is not obtained by a series of points, and that no part of the curve is missing. The success of this instrument is due to its being designed in accordance with the principles of dynamics, and of optics.

The general nature of the apparatus is shown in figures 1 and 2. The success of such an apparatus that is to be free from vibrations of its own is brought about by using an extremely stiff spring. Such a spring is obtained by a short, steel girder, or a circular plate, the girder being shown in

* Contribution from the Ballistic Institute, Clark University No. 4.

figure 1, *c*. The piston *p* is that used in the ordinary gauge gun and bears upon the middle of the girder. From the mass of the piston and the dimensions of the spring we may calculate the natural frequency as about 16,000 per second. It is evident, however, that the damping is so great that no vibrations of the instrument appear. The upper side of the spring is held by two knife edges, on the end of a strong screw held in a massive frame attached to the gun. The arrangement is easy to see from figures 1 and 2. The registration is by means of the image of a spot of light illuminated by an arc lamp and focused by a good photographic lens upon a film carried by a rotating drum, after being reflected upon a small concave mirror attached to the end of the bar.

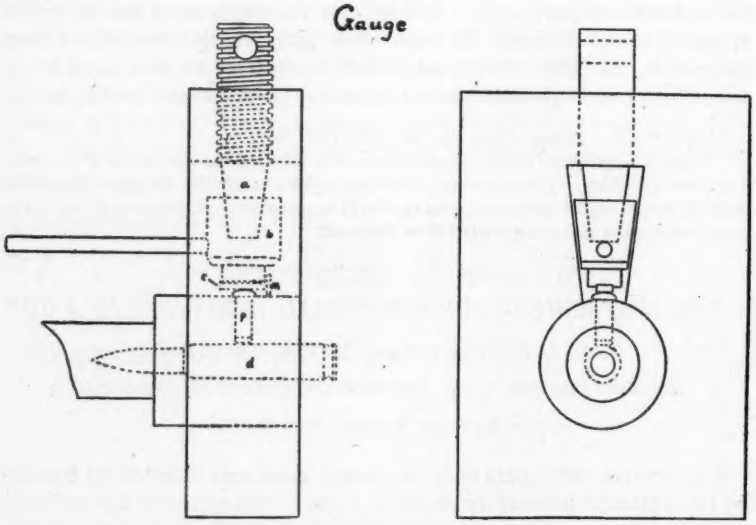


FIG. 1

Instead of rotating the drum it is possible to rotate a totally reflecting prism, so that the film does not need to move. When used to give the *p*, *t* curve (from which the *p*, *s* curve, or ordinary indicator diagram may be obtained by integration—figure 4) the drum is rotated at high speed, say 30 to 50 meters per second peripheral velocity. For maximum pressures it is necessary to move the drum only about 1/100 of a turn as the individual exposures are made. The film when developed shows the series of straight lines the height of which represents the maximum pressure in the barrel. One set of these is shown in figure 3. The information which is available from the *p*, *t* curve is of great importance, and the success of the instrument in giving these curves accurately and with certainty is very gratifying. For example, in

comparing the properties of various powders (which we had occasion to do, see figure 3) the rapidity, duration of burning, maximum pressure, maintenance of pressure, total impulse, and the point and pressure at which the bullet leaves the barrel, and the time in the barrel can all be shown with the $p$, $t$ curve. By mechanical integration we find the curve of velocity and distance and by combining the distance with the pressure we obtain the $p$, $s$ or indicator curve, shown in figure 4. A method is now under experiment to obtain the $p$, $s$ curve directly whether on a small or large gun.

The advantages of the gauge used for maximum pressure measurement are also evident:

1. After the calibration of the spring only one measurement is necessary; namely, the length of the straight line on the film or paper which measures the pressure.

2. The length of this line is usually 50 mm. or more, permitting quick and accurate measurement.

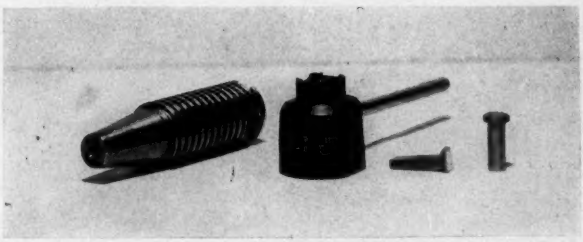

FIG. 2

3. The same instrument is used for every measurement; that is, the flexure of the spring is only temporary. The spring returns absolutely to the zero position there being no permanent set or elastic hysteresis.

4. The operation can be carried out quickly and inexpensively.

5. Permanent record is provided, reading pressures directly on the film or print, and if desired the calibration can be photographed directly on the film.

In order to show experimentally that the static calibration would give accurate results which could not be used in measuring impulsive forces, a calibration was carried out by the use of shock or impulsive forces. The $f$, $t$ curve was obtained on a rotating film as in the ordinary indicator experiment, when a known weight dropped from a known height struck a piston bearing on the spring of the gauge.

In figure 3 are shown a number of curves indicating the properties of different powders furnished by the Winchester Repeating Arms and the DuPont Companies, together with a sample of German anti-tank gun powder furnished by Major Anthony Fiala of the Springfield Armory.

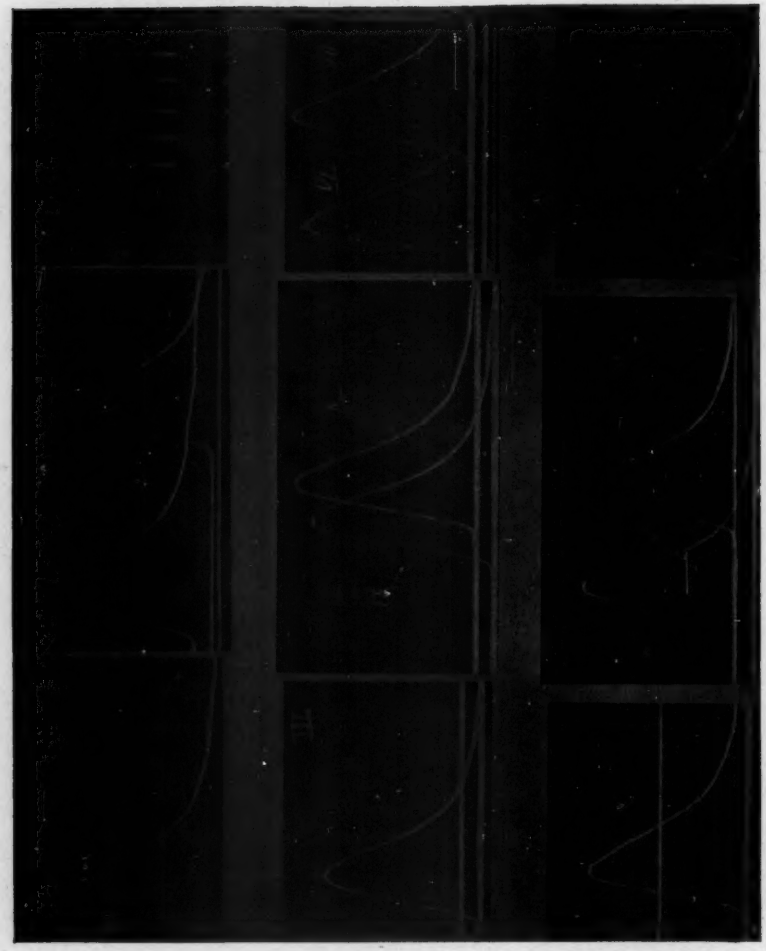

FIG. 3

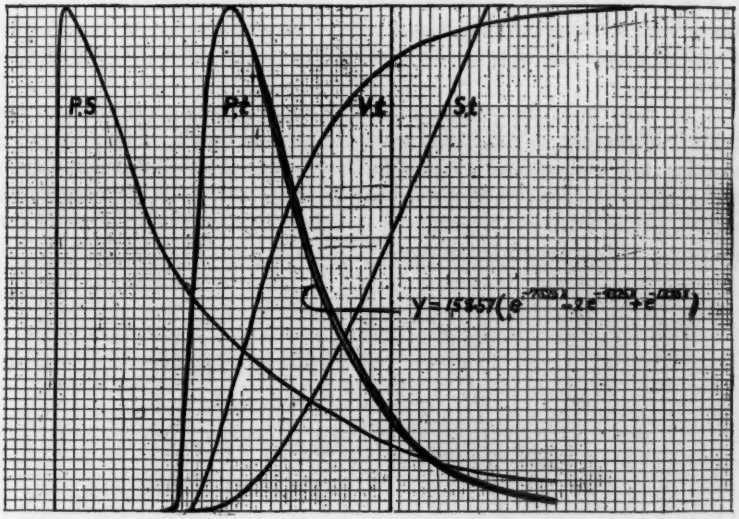


FIG. 4

The emergence of the bullet from the barrel is shown by the interruption of a beam of light thrown upon the film by a separate mirror.

In a subsequent paper the theory of the apparatus and the conclusions that may be obtained from it, including the resistance in the barrel and the variation of specific heats will be described.

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## *ON THE ANGLE OF REPOSE OF WET SAND*

By A. G. Webster

Ballistic Institute, Clark University, Worcester, Massachusetts*

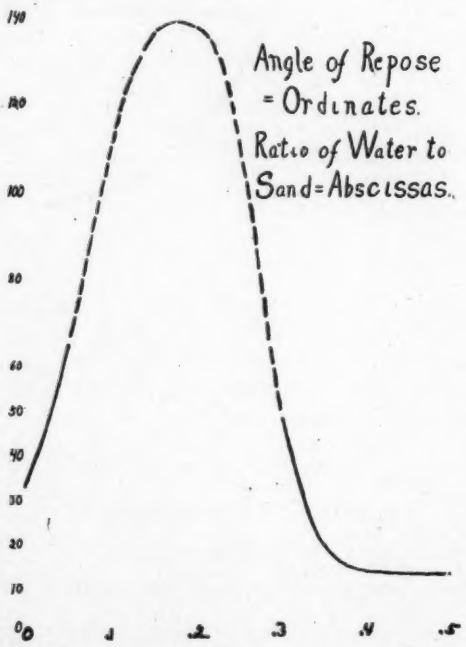
Read before the Academy, April 29, 1919

It is well known that sand, gravel, broken stones, grain, sugar or any pulverulent substance has a definite angle of limiting steepness which is called the angle of repose. In Dr. Breasted's lecture the pile of debris in front of the Temple of Thebes showed such a definite angle. In driving to Washington last week I passed along the Cape Cod Canal and noticed a very definite angle of repose for the sand alongside the Canal. Passing by the beach at Narragansett Pier I measured its slope and found it to be very uniform and about one in fourteen.

*Contribution from the Ballistic Institute, Clark University, No. 6.

A few years ago a committee of this Academy was sent to Panama to examine into the cause of the Culebra slides. My interest was excited at that time, and a year ago by my being consulted as an expert on the collapse of a house.

A few days ago it occurred to me, having a load of sand on the floor of my ballistic laboratory, to wet it and make an artificial beach. I found that on scraping it up with a board a very definite slope was obtained depending upon the wetness. I therefore requested my assistant Dr. E. A. Harrington to make a few quantitative experiments to determine the angle in terms of the wetness. This is shown in the figure.




Beginning with absolutely dry sand which was weighed in a tray a certain amount of water was added and the whole weighed. At first the added water is quickly absorbed and on account of the work done by the surface tension and the cohesion of the water a certain positive amount of cohesion is obtained by the sand, and it will remain in equilibrium vertically and even overhang. Of course these experiments are not extremely accurate. When a certain degree of wetness is passed the sand then acts like a plastic substance, the degree of plasticity depending upon the relative amounts of water and sand.

The status of the mixture is described in the table.

Professor Harry F. Reid informs me that a continental shelf of the Atlantic Ocean has a very definite slope which is very nearly that obtained for the extremely wet sand in these experiments.

I reserve the mathematical theory for a future paper.

| SAND | WATER | ANGLE OF REPOSE | REMARKS |
|---|---|---|---|
| *pounds* | *pounds* | | |
| 10 | 0 | 33° | Dry |
| 10 | 0.5 | 65° | Not hard |
| 10 | 1.0 | 120° | Not accurate, but large obtuse angle, hard |
| 10 | 1.5 | 120°–140° | Not accurate, but large obtuse angle, hard |
| 10 | 2.0 | 120°–140° | Not accurate, but large obtuse angle, hard |
| 10 | 2.5 | 120° | Not accurate, but large obtuse angle, hard |
| 10 | 3.0 | 48° | Fairly hard |
| 10 | 3.5 | 19° | All mixes |
| 10 | 3.75 | 14.5° | Very slight excess of water |
| 10 | 4.0 | 13° | Water not all absorbed |
| 10 | 5.0 | 12° | Excess of water |

The sand has been meshed by Professor Roys of the Worcester Polytechnic Institute with the following results:

| SIZE OF SCREEN | DIAMETER OF OPENING | PER CENT OF SAND WHICH PASSED THROUGH |
|---|---|---|
| *meshes* | *inches* | |
| 200 | 0.0029 | 7.3 |
| 100 | 0.0055 | 29.55 |
| 50 | 0.011 | 84.60 |
| 30 | 0.022 | 99.45 |
| 20 | 0.034 | 99.85 |
| 10 | 0.073 | 99.95 |
| 4 | 0.20 | 100.00 |

## *PALÆOMASTODON, THE ANCESTOR OF THE LONG-JAWED MASTODONS ONLY*

By Henry Fairfield Osborn

American Museum of Natural History, New York City

Read before the Academy, April 29, 1919

In 1900[1] the author predicted that the ancestors of the Proboscideans, as well as of the Hyracoidea and some other orders of mammals, would be discovered in Africa. Two years later the members of the British Geological Survey of Egypt discovered in the Oligocene of the Fayûm remains of *Palæo-*

*mastodon* and of *Mœritherium*, which were at once regarded as the solution of the ancestry of the Proboscideans. These animals took their place in all literature as two steps in the early evolution of this remarkable group.

In 1909[2] Osborn pointed out that *Mœritherium* is to be regarded as a terrestrial form of the Sirenians (manatees and dugongs) in no way directly related to the Proboscideans. It now appears that *Palæomastodon* must also be removed from its generalized position and be regarded as the ancestor of the long-jawed mastodons only; it is far too much specialized in the longirostral direction to be ancestral to the Proboscidea in general. These long-jawed mastodons are distinguished by the peculiar use of the front teeth of the lower jaw, which together made a broadly flattened, spoon-shaped tooth,

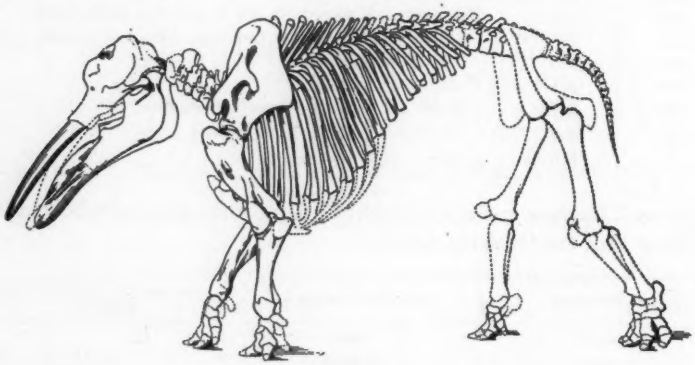


FIG. 1

Outline of the mounted skeleton of *Megabelodon* in The American Museum of Natural History, discovered by E. L. Troxell in the Middle Pliocene of Texas. Drawing one forty-fifth natural size.

almost entirely enamel covered. Phases of the evolution of this long-jawed phylum are seen in the classic *Trilophodon angustidens* of Cuvier, in the lower Miocene of France. A branch reached Texas in the Upper Miocene (*Trilophodon productus* of Cope), and Florida as well as Texas in the *Trilophodon serridens* of Cope. It attained gigantic proportions in the Middle Pliocene. The *Megabelodon* of Barbour, a superb skeleton of a long-jawed and extremely short-limbed Proboscidean, recently discovered in Texas by Mr. E. L. Troxell, has been mounted in The American Museum of Natural History. It represents one of the culminating stages in the evolution of the long-jawed mastodons. In these animals we find proof of nearly direct linear descent from the *Palæomastodon* of the Fayûm, e.g., the long enamel band on the upper tusks, the broadly spoon-shaped arrangement of the lower tusks with enamel covering. In massiveness these animals parallel and even surpass the true mastodons of the Pleistocene, to which they are only indirectly related.

[1] Osborn, 1900, 182. [2] Osborn, 1909, 332.

# THE ELONGATION DUE TO MAGNETIZATION

By C. Barus

Department of Physics, Brown University

Communicated, April 30, 1919

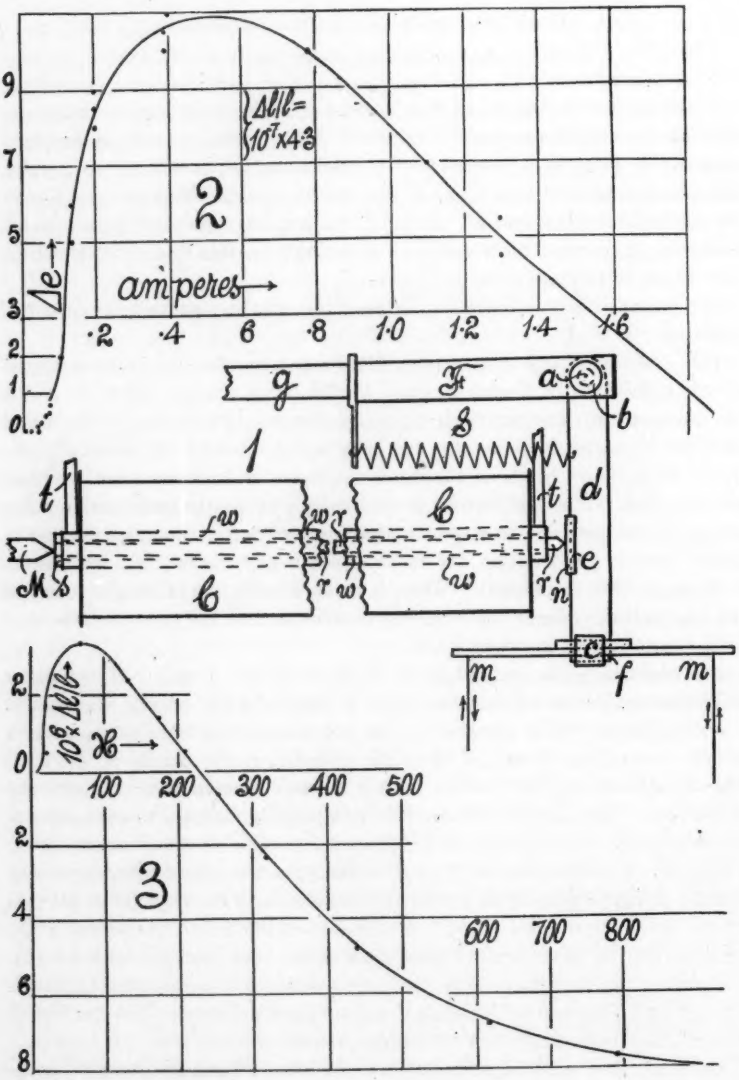
*1. Introductory.*—The small longitudinal displacements due to magnetostriction have been frequently subjected to investigation and an excellent summary is given in Winkelmann's Handbuch, vol. 5, p. 307, et seq., 1908. The measurements of Prof. C. G. Knott and his students Nagaoka and Honda are particularly noteworthy.[1] In 1911[2] my son, Mr. Maxwell Barus, and I used these phenomena for the purpose of testing a peculiar type of interferences then under discussion.

The present purpose is similar, being a test of the contact lever recently[3] described.

The elongation (and contraction) phenomena are necessarily complicated by the occurrence of hysteresis loops to which the present paper (in which the measurements are not made by the continuous variation of currents and field, but by successively making and breaking the circuit) will give no attention. This subject has been adequately explored by Professor Knott and the authors cited. The chief interest in this paper is rather the continued increase of the contractions due to magnetization, not only after the latter has practically reached a maximum, but in a marked degree (so far as I have gone, fields up to 800), indefinitely. There is no sure indication of an abatement of the contraction. Hence the magnetic contribution of the present paper is to lie in the treatment in strong fields.

*2. Apparatus.*—The contact lever shown in figures 1 and 2 of the paper cited was modified as indicated in figure 1, where $F$ is the semicircular fork in a vertical plane, rigidly attached to the bed plate of the interferometer by a strong clutch (not shown), holding the cylinder, $g$, the handle of the fork. The vertical axis $a$ of the contact lever is secured between the screw pivots $b$ of the fork. The horizontal strip of brass $d$, rigidly fastened to the center of the axle $a$, carries at its end the auxiliary mirror $mm'$ of the quadratic interferometer. For this purpose, a short length, $f$, at the end of $d$ has been bent upward at right angles to $d$, so that $mm'$ may be held between plates of brass by the yoke-shaped steel clip $c$. At the side of the lever is a vertical brass plate inset $c$, to which a small glass plate $n$ has been fastened with cement. It is against this that the conical end of the iron rod $rr$ to be examined, pushes. The spring $S$ attached to the blade $d$ and a lateral projection from the fork $F$ insures continuous contact at a constant pressure.

The iron rod $rr$ originally about 43 cm. long and 67 cm. in diameter, is enveloped by a tubular water jacket $ww$. Through this a current of water entering at $t$ and leaving at $t'$ is kept flowing from a large copper Mariotte flask about 50 cm. high and 30 cm. in diameter, and containing water at the tempera-


Δl/l=
10⁻⁷x4∂
2
amperes
Δe
·2 ·4 6 ·8 1·0 1·2 1·4 1·6
9 7 5 3 2 1 0
g F a b
1
t w r b M s
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100 200 300 400 500 600 700 800
3
2 0 2 4 6 8

ture of the room. Two such flasks were at hand to be used alternately. The coil *CC* 26 cm. long and 3.7 cm. in external diameter, is wound immediately on the tubular water jacket.

The rod *rr* fits the tube *ww* loosely and is centrally detached at the remote end by aid of a bushing *s* and a small bolt. The front end is free. The coil *CC* is held in position by a large clutch (not shown) encircling it at the middle and attached to the bed plate of the interferometer. It is additionally attached at the tubulures $t$ and $t'$. Finally the conical end $M$ of a micrometer screw (also rigidly attached to the bed plate) gives the remote end of the rod *rr* any desirable fiducial position. This micrometer $M$ has the further advantage of permitting an independent standardization of the contact lever, as there is obviously always sufficient elastic yielding in the apparatus to considerably shift the interference fringes.

In the experiments made, the breadth of the ray rectangle $mm'$ of the interferometer was $b = 9.7$ cm.; the normal distance between the rod *rr* and the axis $a$, 7 cm.; the length of the contact lever $a$ to $mm'$, 10.6 cm.; and the length of the axle $a$, 10 cm.

*3. Observations.*—The helix $C$ in figure 1 was slender in shape, the length being 37 cm. and the diameter within being about 1.5 cm. There were about 11.2 turns per centimeter per layer and 8 layers of wire so that the field within may be estimated at $H = 110\, i$ gauss, $i$ being the current in amperes. The current 0.01 to over 8 amperes thus corresponded to field from 1 to over 800 gauss.

The first rod selected was of low carbon shop steel 43 cm. long, so that it projected a few centimeters beyond either end of the helix.

The displacement of fringes observed was characteristic, being (in the smaller fields) slow and deliberate on closing the circuit (so that their motion could almost be followed by the eye), but very rapid on breaking the circuit.

The experiments were begun with small fringes (about 0.1 mm. in the ocular), and the readings $\Delta e$ were made in terms of an ocular micrometer scale which was a centimeter divided into 0.1 mm. This was compared with the datum, $\Delta N$, of the displacement micrometer normal to one of the mirrors of the interferometer.

If $\Delta N$ corresponds to the angular displacement, $\Delta\theta$, of the contact lever and to $\Delta l$ of elongation of the iron rod $r$ in the helic $C$ (figure 1) we may write as above

$$2b\Delta\theta = 2\Delta N \cos i \tag{1}$$

if $i$ is the angle of incidence (45°) at the mirrors of the interferometer and $b$ the breadth of the ray parallelogram. But

$$\Delta l = r\Delta\theta \tag{2}$$

if $r$ is the normal distance of the line of thrust of the rod *rr* from the axis of the contact lever. Thus

$$\Delta l = (r \cos i/b)\ (\Delta N/\Delta e)\ \Delta e \tag{3}$$

If $l$ is the length of the iron rod $\Delta l/l$ will be the datum required.

It was quite possible to ascertain $\Delta e = 0.1$; i.e., elongations of but $\Delta l/l = 5 \times 10^{-8}$, equivalent to $\Delta l = 2.3 \times 10^{-5}$ cm. The current must exceed 0.02 am. before any elongation can be detected, after which, however, the elongations abruptly begin and increase rapidly to a maximum, which is reached before saturation.

The experiments, figure 2, were made with somewhat greater care and with larger fringes. The standardization of the ocular micrometer showed

$$\Delta N/\Delta e = 10^{-5} \times 1.82, \quad \Delta l/l = 10^{-7} \times 2.16\, \Delta e$$

But for incidental difficulties (tremors, etc,), the results in figure 2 would probably be very smooth.

A number of supplementary experiments (see figure 2) were made to see whether the observed $\Delta l = 0$ for currents below 0.02 amperes might not be equivalent to an initial small minimum. But $\Delta l$ remained persistently zero, while currents decreased from 0.02 to 0.001 amperes. At 0.004 am. the field was reversed, but no significant $\Delta l$ could be observed. The fringes just moved ($\Delta e = 0.1$) when $i$ was about 0.035 amperes, indicating a field of 3 or 4 gauss.

A rough test made of the equation by pushing the rod *rr* forward by the backstop screw $M$, figure 1, gave corroborative results.

If $\Delta\varphi$ refers to the turns of the screw $M$

$$\Delta l = (r \cos i/b)\, (\Delta N/\Delta e)\, (\Delta e/\Delta\varphi)\, \Delta\varphi$$

$(\Delta N/\Delta e)$ was found to be $10^{-5} \times 3.3$ cm. per scale part and $(\Delta e/\Delta\varphi)$ scale parts per degree of turn. Hence with the above data

$$\Delta l = 10^{-4} \times 1.8$$

The back screw having 40 turns to the inch, i.e., a pitch of 0.0635 cm. gives us $10^{-4} \times 1.76$ cm. per degree of turn.

Another feature may be mentioned here. The expansion of the coil when carrying very large currents is a thrust on the back stop $M$, which is quite appreciable and appears as an apparent contraction of the rod.

*4. Vibration telescope.*—To test the surmise that initial elongations always precede the final contraction, the vibration telescope heretofore described[4] was installed. It was then found that the even band of fringes drawn out by the vibrating objective broke up into strongly sinuous lines on making and particularly on breaking the circuit through the helix. When the circuit was made and broken alternately, the waves broke up further into a succession of discontinuous pulses of more than double the amplitudes of the waves. With the field properly adjusted by passing 1.8 to 2 amperes through the coil, there was no further observable displacement after the strong waves lines, produced immediately after closing the circuit, had subsided.

*5. Further observations.*—The data of figure 3 were investigated in a single series. To reduce the heat discrepancy a brisk current of water was passed through the tubular water jacket. This seemed the safer plan, even though the fringes were shaken. The observations were made in triplets and largely confined to the higher fields.

The curve is quite as clearly indicated as may be expected owing to the difficulties cited; but the higher observations ($H > 200$) are decreasing contractions. The reason of this is partly owing to the method of observation in triplets, where (curiously enough) the third reading (field zero) was a contraction in relation to the first reading in the absence of the field.

The apparatus in these experiments was therefore suspected of being faulty in design, inasmuch as the clutch of the contact lever and of the coil were attached to the same rigid standard. This arrangement was now modified, so that the two mountings were quite independent, whereupon the anomalous results specified largely receded.

As a second test a rod 28 cm. long of Swedish iron was inserted, the extra length being pieced out by brass tubing soldered to each end, so that the iron lay quite within the coil. The data obtained closely resembled figure 3. Tests made with other metals gave no positive results.

*6. Coefficient of expansion.*—To arrive at a definite reason for the occurrence of the anomalous contraction mentioned above, it seemed desirable to modify the magnetic apparatus for the purpose of measuring the coefficient of expansion of a given metal. This could easily be done by using the coil merely as a heater.

A number of experiments were made, using either the ocular micrometer (here the temperature increments must lie within 2°) and the Fraunhofer micrometer at the mirror of the interferometer. It was observed that all the expansions were apt to begin with a contraction immediately after the heating current had been closed. Hence there is an initial expansion of the coil itself. It was soon found that the consequent flexure of the table was the ultimate cause of the interferometer discrepancy.

A modification of the apparatus was made therefore by allowing the end $C$ of the coil to recline on a large grooved wheel, which by slight rotation would admit of any expansion of the kind in question. With this improvement the anomalous contractions vanished and the work thereafter proceded smoothly.

*7. Theoretical observations.*—To account for such a graph as figure 3, as a whole, one may argue that the initial elongations are to be referred to the rotations of the molecular magnets. For these elongations are coextensive in field variations with the marked increase of magnetization. It would then seem plausible that thereafter the attractions between the oriented molecular magnets may be instanced to account for the persistent contractions in continually increasing fields. Thus it seems worth while to endeavor to ascertain whether such a supposition would conform with any reasonable value of the susceptibility $k$ of the iron, which one may estimate as decreasing from over

100 to less than 10 as the rod approaches saturation ($H = 150$ gauss) and to decrease thereafter asymptotically to zero.

If $p$ is the force per square centimeter of section of the rod and $E$ Young's modulus,

$$p = E(\Delta l/l) \tag{1}$$

regarding the magnetic stress as traction.

Using the expression for the potential of a disc, the field $F$ in a narrow crevasse normal to $F$, between molecular layers of magnetic surface density of magnetization $kH$

$$F = 4\pi kH + H$$

where $k$ is the susceptibility of the metal.

Hence the force per square centimeter should be $p' = FkH$, or

$$p' = H^2(4\pi k^2 + k) \tag{2}$$

Equating $p = p'$ in equations (1) and (2)

$$\frac{\Delta l}{l} = \frac{4\pi k^2 + k}{E} H^2 \tag{3}$$

If the data in figure 3 are taken above 800 gauss, supposing that these are far enough removed from the initial complications, the estimate would be ($E = 2 \times 10^{12}$), $k = 1.6$.

An order of mean susceptibility of 1.6 (which seems not an unreasonable assumption) would thus account for the observed contractions. Naturally as $k$ is essentially variable with $H$ a better statement of the case might be given by postulating such a relation.

[1] *London, Phil. Mag.*, 37, 1894, (131).
[2] *Carnegie Inst., Washington, Pub.*, No. 149.
[3] These PROCEEDINGS, 5, 1919, (39).
[4] These PROCEEDINGS, 4, 1918, (328).

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## *GROUPS INVOLVING ONLY TWO OPERATORS WHICH ARE SQUARES*

BY A. G. MILLER

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ILLINOIS

Communicated by E. H. Moore, May 2, 1919

The abelian group of order $2^m$ and of type $(1, 1, 1, \ldots)$ is completely characterized by the fact that all of it operators have a common square. When we impose the condition that the operators of a finite group $G$ have two and only two distinct squares then $G$ must belong to one of three infinite systems of groups whose characteristic properties we proceed to determine.

All the operators of $G$ besides the identity must be of order 2 or of order 4 and $G$ must involve operators of each of these two orders. Hence the order of $G$ is of the form $2^m$. When $G$ is abelian it is of type (2, 1, 1, . . . ) and it will therefore be assumed in what follows that $G$ is non-abelian. The octic group and the quaternion group constitute well known illustrations of such a group and have the smallest possible order.

When the operators of order 2 contained in $G$ together with the identity constitute a subgroup this subgroup is the central of $G$ and hence $G$ belongs to the system of groups called Hamiltonian by R. Dedekind.[1] In this case it is known that $G$ is the direct product of the quaternion group and an abelian group of order $2^\alpha$ and of type (1, 1, 1, . . . ). Hence it will be assumed in what follows that $G$ involves non-commutative operators of order 2.

Every operator of order 4 contained in $G$ is transformed either into itself or into its inverse by every operator of $G$ and an operator of order 2 contained in $G$ has at most two conjugates under the group.[2] Let $H_1$, $H_2$ represent subgroups composed respectively of all the operators of $G$ which are commutative with two non-commutative operators of order 2 $s_1$, $s_2$. The cross-cut $K_1$ of $H_1$ and $H_2$ is of index 4 under $G$ and includes the central of $G$. A set of independent generators of $G$ can be so selected as to include $s_1$, $s_2$ and operators from $K_1$.

Exactly one-half of the operators of $G$ which are not also in $K_1$, are of order 2 since the quotient group $G/K_1$ is abelian. If $K_1$ involves non-commutative operators of order 2 two such operators $s_3$, $s_4$ may be selected from $K_1$ in exactly the same way as $s_1$ and $s_2$ were selected from $G$. The remaining operators of a set of independent generators including $s_1$, $s_2$, $s_3$, $s_4$ may be selected from an invariant subgroup of index 4 under $K_1$ and of index 16 under $G$ all of whose operators are commutative with each of the four operators already chosen.

As $G$ is supposed to be of finite order we arrive by this process at a subgroup $K_m$ in which all the operators of order 2 are commutative. Hence $K_m$ belongs to one of the following three well known categories of groups. Abelian and of type (1, 1, 1, . . . ), abelian and of type (2, 1, 1, . . . ), or Hamiltonian of order $2^a$. The commutator subgroup of $G$ is of order 2.

In each case, $G$ may be constructed by starting with $K_m$, forming the direct product of $K_m$ and an operator $t_1$ of order 2, and then extending this direct product by means of an operator $t_2$ of order 2 which is commutative with each of the operators of $K_m$ and transforms $t_1$ into itself multiplied by the commutator of order 2 contained in $G$. When $K_m$ is Hamiltonian or abelian and of type (2, 1, 1, . . . ) this commutator is determined by $K_m$. In the other possible case it may be selected arbitrarily from the operators of order 2 found in $K_m$.

When $m>1$, we use the group $K_{m-1}$ just constructed in exactly the same way as $K_m$ was used in the preceding paragraph. The commutator of order 2 is completely determined for each of the categories by $K_{m-1}$, $m>1$. When $m>2$

we proceed in the same manner with $K_{m-2}$, etc. It may be noted that in each of the groups belonging to one of the three categories thus constructed more than one-half of the operators are of order 2, in those belonging to the second category the number of operators of order 2 is one less than one-half of the order of $G$, while in those belonging to the third category the number of operators of order 2 is obtained by subtracting from one-half the order of $G$ one plus one-fourth the order of $K_m$.

Some of these results constitute a proof of the following theorem: *If only two of the operators of a group G are the squares of operators contained in G then the non-invariant operators of G have only two conjugates, each cyclic subgroup of order 4 is invariant, and G belongs to one of three categories of groups of order 2 which can be separately generated by a set of operators such that each of these operators is commutative with each of the others except at most one of them.*

When $m$ is sufficiently large there is one and only one group belonging to each of these three categories and having a give number $\gamma$ of pairs of non-commutative operators of order 2 in its set of independent generators when this set is obtained in the manner described above. The smallest values of $m$ for these categories are $2\gamma + 1$, $2\gamma + 2$, and $2\gamma + 3$ respectively. When $m$ has a larger value $G$ must be the direct product of an abelian group of type (1, 1, 1, . . . ) and of the minimal group having $\gamma$ such pairs of generators and contained in the category to which $G$ belongs.

By means of these facts it is very easy to determine the number of the groups of a given order $2^m$ which belong to each of these three categories. This number is the largest integer which does not exceed $\frac{m-1}{2}$, $\frac{m-2}{2}$, and $\frac{m-3}{2}$ for the three categories respectively. In particular, the number of the distinct groups of order 128 belonging to each of these categories is 3, 2, 2 respectively, it being assumed that each of the groups in question contains at least two non-commutative operators of order 2.

In each one of these groups every two non-commutative operators of order 2 generate the octic group and every two non-commutative operators of order 4 generate the quaternion group. Moreover, every non-abelian subgroup is invariant. In two of the categories the central is composed of operators of order 2 in addition to the identity, while the central of the remaining category is of type (2, 1, 1, . . . ). Every one of these groups is generated by its operators of order 2. From the standpoint of definition and structure these categories rank among the simplest known infinite systems of non-abelian groups

[1] Dedekind, R., *Math. Ann., Leipzig*, **48**, 1897, (548–561).

[2] Miller, G. A., *Trans. Amer. Math. Soc., New York*, **8**, 1907, (1–13).

# ACOUSTICAL IMPEDANCE, AND THE THEORY OF HORNS AND OF THE PHONOGRAPH

By Arthur Gordon Webster

Department of Physics, Clark University

Communicated, May 8, 1919*

The introduction more than thirty years ago of the term 'impedance' by Mr. Oliver Heaviside has been productive of very great convenience in the theory of alternating currents of electricity. Unfortunately, engineers have not seemed to notice that the idea may be made as useful in mechanics and acoustics as in electricity. In fact, in such apparatus as the telephone one may combine the notions of electrical and mechanical impedance with great advantage. Whenever we have permanent vibrations of a single given frequency, which is here denoted, as usual, by $n/2\pi$, the notion of impedance is valuable in replacing all the quantities involved in the reactions of the system by a single complex number. If we follow the convenient practice of denoting an oscillating quantity by $e^{int}$ and taking its real part (as introduced by Cauchy) all the derivatives of $e^{int}$ are obtained by multiplication by powers of $in$, or graphically by advancing the representative vector by the proper number of right angles.

If we have any oscillating system into which a volume of air $X$ periodically enters under an excess pressure $p$, I propose to define the impedance by the *complex* ratio $Z = p/X$. If we call $dX/dt = I$ the current as in electricity, if we followed electrical analogy we should write $Z = pI$ so that the definition as given above makes our impedance lead by a right angle the usual definition. I believe this to be more convenient for our purposes than the usual definition and it need cause no confusion.

If we have a vibrating piston of area $S$ as in the phonometer, we shall refer its motion to the volume $S\xi$ it carries with it and the force acting on it to the pressure, so that $F = Sp$. The differential equation of the motion is

$$m\frac{d^2\xi}{dt^2} + \kappa\frac{d\xi}{dt} + f\xi = F = Sp, \qquad X = S\xi, \tag{1}$$

we have

$$Z_1 = (f - mn^2 + i\kappa n)/S^2, \tag{2}$$

where $m$ is the mass, $\kappa$ the damping, $f$ the stiffness. The real part of $S^2Z$, $f - mn^2$, is the uncompensated stiffness, which is positive in a system tuned too high, when the displacement lags behind the force, by an angle between zero and one right angle, negative when the system is tuned too low, when the

* This article was read in December 1914 at the meeting of the American Physical Society at Philadelphia, and has been held back because of the continual development of the experimental apparatus described in a previous paper in these Proceedings.

lag is between one and two right angles, as shown in figure 1. If we force air into a chamber of volume $V$, the compression $s = X/V$ will be related to the excess pressure $p$ by the relation $p = es$, where $e$ is the modulus of elasticity of the air $e = \rho a^2$, $\rho$ being the density and $a$ the velocity of sound. Consequently we have

$$Z_0 = \frac{e}{V} = \frac{\rho a^2}{V}, \tag{3}$$

and the analogy is to a condenser. If we have air passing through an orifice or short tube of conductivity $c$ its inertia gives an apparent mass $\rho/c$, and if it

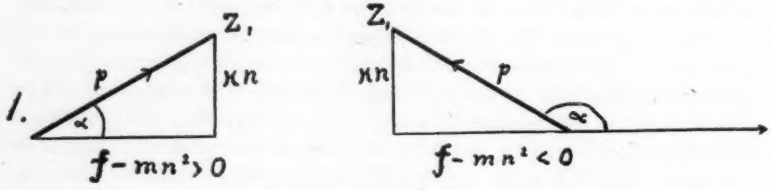



FIG. 1

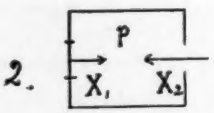



FIG. 2

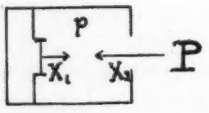



FIG. 3

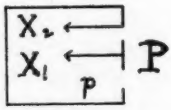



FIG. 4

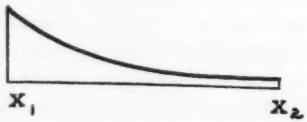



FIG. 5

escapes from a circular hole in an infinite plane it dissipates energy so that the whole impedance credited to the hole is

$$Z_2 = -\frac{\rho n^2}{c} + \frac{\rho n^3}{2\pi a}\, i = ek^2 \left\{ \frac{k}{2V}\, i - \frac{1}{c} \right\}, \text{ where } k = \frac{n}{a}. \tag{4}$$

These three typical impedances will be at constant use in acoustics. It is to be remembered that systems in series have their impedances added and in parallel have the reciprocals of impedance added. Also that the free vibrations of a system are obtained by equating the impedances to zero.

As a simple example consider the phone described in the previous article, figure 3.

Let $X_1 = S\xi$ be the volume introduced by the piston $X_2$ that entering by the hole. Then

$$p = Z_0 (X_1 + X_2) = - Z_2 X_2, \tag{5}$$

$$X_2 = \frac{- Z_0}{Z_0 + Z_2} X_1,$$

and inserting values,

$$X_2 = - \frac{S\xi}{1 + Vk^2 \left( \frac{ki}{2\pi} - \frac{1}{c} \right)} \tag{6}$$

Disregarding phase by taking the modulus and putting $k = n/a$ we have the phone formula for the strength of source.

$$A = \left| \frac{dX}{dt} \right| = S \Psi \, | \, \xi \, |, \tag{7}$$

where

$$\Psi = \frac{n}{\left\{ \left( 1 - \frac{Vk^2}{c} \right)^2 + \frac{V^2 k^6}{4\pi^2} \right\}^{\frac{1}{2}}} \tag{8}$$

If instead of sending the air out through a hole it goes into a cone or any other horn, we must use for the impedance $Z_2$ that given below, and we arrive at the theory of the phonograph, and are thus able to answer the question as to the function of the horn in persuading the sound to come out of the phonograph when the motion of the diaphragm is given (it is well known that very little sound is emitted by the phonograph or the telephone with the horn taken off, although in the former case the motion of the diaphragm is exactly the same).

The phonometer was formerly arranged with the back of the diaphragm protected from the sound, figure 3. Let $P$ be the external pressure, then, as before,

$$p = Z_0 (X_1 + X_2) \tag{9}$$

and in addition,

$$\begin{aligned} -p &= Z_1 X_1, \\ P - p &= Z_2 X_2, \end{aligned} \tag{10}$$

from which

$$X_1 = \frac{- PZ_0}{Z_0 Z_1 + Z_1 Z_2 + Z_2 Z_0}, \tag{11}$$

giving the formula for the measurement of the pressure,

$$P = \varphi \xi / S \tag{12}$$

$$\varphi = \frac{\gamma}{[\{u\nu - (\alpha\beta + \gamma^2)\}^2 + \{\beta u + \alpha\nu\}^2]^{\frac{1}{2}}} \tag{13}$$

and $\varphi$ may be termed the sensitiveness of the phonometer. Where

$$\gamma = S^2 Z_0 = S^2 \rho a^2 / V, \qquad \alpha = \kappa n, \qquad \beta = S^2 \rho n^3 / 2\pi a,$$

$$u = f - n^2 m + S^2 \rho a^2 / V, \qquad v = S^2 \{\rho a^2 / V - \rho u^2 / c\} = S^2 Z_0 (1 - k^2 V / c) \quad (14)$$

As described in my recent article the back of the piston is exposed to the sound, figure 4. Then

$$\begin{aligned} P - p &= Z_1 X_1 = Z_2 X_2 \\ p &= Z_0 (X_1 + X_2) \end{aligned} \quad (15)$$

from which

$$X_1 = \frac{Z_2 P}{Z_0 Z_1 + Z_1 Z_2 + Z_2 Z_0} \quad (16)$$

$$\varphi = \left[ \frac{(v - \gamma)^2 + \beta^2}{\{uv - (\alpha\beta + \gamma^2)\}^2 + (\alpha v + \beta u)^2} \right]^{\frac{1}{2}}, \quad (17)$$

*Tubes and Horns.*—Beside the above described phone and phonometer, the theory of which assumed a resonator so small that the pressure is supposed to be the same at every internal point, I have made use of many arrangements employing tubes or cones, in which we must take account of wave-motion. The familiar theory of cylindrical pipes may be included in the following generalized theory, which I have found experimentally to serve well.

Let us consider a tube of infinitesimal cross section $\sigma$ varying as a function of the distance $x$ from the end of the tube. Then if $q$ is the displacement of the air, $p$ the pressure, $s$ the compression, we have the fundamental equations

$$p = es = \rho a^2 s = -e \operatorname{div} q = -\frac{e}{\sigma} \frac{d(q\sigma)}{dx}, \quad (18)$$

$$\frac{d^2 p}{dt^2} = a^2 \Delta p = a^2 \operatorname{div} \operatorname{grad} p = a^2 \left\{ \frac{1}{\sigma} \frac{d}{dx} \left( \sigma \frac{dp}{dx} \right) \right\} \quad (19)$$

$$\frac{d^2 q}{dt^2} = -\frac{1}{\rho} \frac{dp}{dx} = a^2 \frac{d}{dx} \left\{ \frac{1}{\sigma} \frac{d}{dx} (q\sigma) \right\}. \quad (20)$$

For a simple periodic motion we put $p$, $q$ proportional to $e^{int}$, and obtain

$$\frac{d^2 p}{dx^2} + \frac{d \log \sigma}{dx} \frac{dp}{dx} + k^2 p = 0, \quad \frac{d^2 q}{dt^2} + \frac{d \log \sigma}{dx} \frac{dq}{dx} + \frac{d^2 \log \sigma}{dx^2} q + k^2 q = 0. \quad (21)$$

Both these linear equations may be solved by means of series, and if we call $u(kx)$, $v(kx)$ two independent solutions we have

$$p = Au + Bv, \quad \beta q = Au' + Bv', \quad \beta = \rho a^2 k,$$

where the accents signify differentiation according to $kx$. If we denote values at one end $x = x_1$ and at the other end $x = x_2$ by suffixes 1, 2, respectively, and form the determinants

$$D_1 = \begin{vmatrix} u_1, v_1 \\ u_1, v_1 \end{vmatrix}, \quad D_2 = \begin{vmatrix} u_2, v_2 \\ u_2, v_2 \end{vmatrix}, \quad D_3 = \begin{vmatrix} u_1, v_1 \\ u_2, v_2 \end{vmatrix},$$
$$D_4 = \begin{vmatrix} u_2, v_2 \\ u_1, v_1 \end{vmatrix}, \quad D_5 = \begin{vmatrix} u_1, v_1 \\ u_2, v_2 \end{vmatrix}, \quad D_6 = \begin{vmatrix} u_1, v_1 \\ u_2, v_2 \end{vmatrix}, \tag{22}$$

which satisfy the relation,

$$D_1 D_2 = D_3 D_4 + D_5 D_6$$

we may determine the constants $A, B$ in terms of any two out of $p_1, q_1, p_2, q_2$, so that we obtain

$$\begin{aligned} p_2 &= (p_1 D_4 + \beta q_1 D_5)/D_1, & \beta q_2 &= (-p_1 D_6 + \beta q_1 D_3)/D_1, \\ p_1 &= (p_2 D_3 - \beta q_2 D_5)/D_2, & \beta q_1 &= (p_2 D_6 + \beta q_2 D_4)/D_2. \end{aligned} \tag{23}$$

As it is more convenient to deal with the volumes $X_1 = \sigma_1 q_1$, $X_2 = \sigma_2 q_2$ we shall have in general

$$p_2 = a p_1 + b X_1, \quad X_2 = c p_1 + d X_1, \tag{24}$$

where

$$a = \frac{D_4}{D_1}, \quad b = \frac{\beta}{\sigma_1}\frac{D_5}{D_1} \quad c = -\frac{\sigma_2}{\beta}\frac{D_6}{D_1}, \quad d = \frac{\sigma_2}{\sigma_1}\frac{D_3}{D_1}, \quad ad - bc = \frac{\sigma_2}{\sigma_1}\frac{D_2}{D_1},$$

and for the impedances belonging to the ends of the tube

$$Z_2 = \frac{aZ_1 + b}{cZ_1 + d_1}, \qquad Z_1 = \frac{dZ_2 - b}{-cZ_2 + a}, \tag{25}$$

so that the impedance at either end of the tube is a linear fractional function of the other. According to the apparatus attached to an end the impedance attached to that end is known. A tube for which $a, b, c, d$ are given may be replaced by any other tube having the same constants.

*Examples.*—Cylindrical tube, $\sigma$ constant. Put $x_2 - x_1 = Z_1$,

$$\frac{d^2 p}{dx^2} + k^2 p = 0,$$

$$u = \cos kx, \qquad r = \sin kx, \qquad u' = -\sin kx, \qquad v' = \cos kx, \tag{26}$$

$$D_1 = D_2 = 1, \qquad D_3 = D_4 = \cos kl, \qquad D_5 = D_6 = \sin kl, \tag{27}$$

$$a = d = \cos kl, \qquad b = \frac{\beta}{\sigma}\sin kl, \qquad c = -\frac{\sigma}{\beta}\sin kl,$$

$$Z_2 = \frac{\beta}{\sigma}\frac{Z_1 \cos kl + \frac{\beta}{\sigma}\sin kl}{-Z_1 \sin kl + \frac{\beta}{\sigma}\cos kl}, \quad Z_1 = \frac{\beta}{\sigma}\frac{Z_2 \cos kl - \frac{\beta}{\sigma}\sin kl,}{Z_2 \sin kl + \frac{\beta}{\sigma}\cos kl} \tag{28}$$

Conical tube, $\sigma = \sigma_0 x^2$

$$\frac{d^2p}{dx^2} + \frac{2}{x}\frac{dp}{dx} + k^2 x = 0 \tag{29}$$

$$u = \frac{\cos kx}{kx}, \quad v = \frac{\sin kx}{kx,} \quad u' = -\left(\frac{\sin kx}{kx} + \frac{\cos kx}{k^2x^2}\right), \quad v' = \frac{\cos kx}{kx} - \frac{\sin kx}{k^2x^2}$$

$$D_1 = \frac{1}{k^2x_1^2}, \quad D_2 = \frac{1}{k^2x_2^2}, \quad D_3 = \frac{\cos kl}{k^2x_1x_2},$$

$$D_4 = \frac{\cos kl}{k^2x_1x_2} + \frac{\sin kl}{k^3x_1x_2^2}, \quad D_5 = \frac{\sin kl}{k^2x_1x_2}$$

and if we introduce two lengths $\epsilon_1$, $\epsilon_2$, defined by the equations

$$\tan k\epsilon_1 = kx_1, \qquad \tan k\epsilon_2 = kx_2,$$

we easily get

$$a = \frac{x_1}{x_2}\frac{\sin k(l+\epsilon_1)}{\sin k\epsilon_1}, \qquad b = \frac{\beta}{\sigma_1}\frac{x_1}{x_2}\sin kl, \tag{30}$$

$$c = -\frac{\sigma_2}{\beta}\frac{x_1}{x_2}\frac{\sin k(l+\epsilon_1-\epsilon_2)}{\sin k\epsilon_1 \sin k\epsilon_2}, \qquad d = \frac{\sigma_2}{\sigma_1}\frac{x_1}{x_2}\frac{\sin k(l-\epsilon_2)}{\sin k\epsilon_2}, \tag{31}$$

$$Z_2 = -\frac{\beta}{\sigma_2}\frac{Z_1\dfrac{\sin k(l+\epsilon_1)}{\sin k\epsilon_1} + \dfrac{\beta}{\sigma_1}\sin kl,}{Z_1\dfrac{\sin k(l+\epsilon_1-\epsilon_2)}{\sin k\epsilon_1 \sin k\epsilon_2} + \dfrac{\beta}{\sigma_1}\dfrac{\sin k(l-\epsilon_2)}{\sin k\epsilon_2}}$$

$$Z_1 = -\frac{\beta}{\sigma_1}\frac{Z_2\dfrac{\sin k(l-\epsilon_1)}{\sin k\epsilon_2} + \dfrac{\beta}{\sigma_2}\sin kl}{Z_2\dfrac{\sin k(l+\epsilon_1-\epsilon_2)}{\sin k\epsilon_1 \sin k\epsilon_2} + \dfrac{\beta}{\sigma_2}\dfrac{\sin k(l+\epsilon_1)}{\sin k\epsilon_2}} \tag{32}$$

The formulae (31), (32) were used by Professor G. W. Stewart in designing horns to be used during the war.

It is not true, as is frequently stated in books on musical instruments, that the brass instruments of the orchestra are hyperbolic in profile, but I have found for all practical purposes the bell of every instrument may be represented by one of the three formulae

$$\sigma = \sigma_0 x^n, \qquad \sigma = \sigma_0 e^{-mx}, \qquad \sigma = \sigma_0 e^{-mx^2}$$

Even if an equation cannot be given to the profile the differential equation may be easily integrated graphically, or the length may be divided up into sections and different values of $n$ used for different sections, as is customary in the theory of ballistics.

Case 1. $\sigma = \sigma_0 x^n$. (Change units so that $k = 1$)

$$\frac{d^2p}{dx^2} + \frac{n}{x}\frac{dp}{dx} + p = 0, \qquad \frac{d^2X}{dx^2} - \frac{n}{x}\frac{dX}{dx} + X = 0 \tag{34}$$

We have

$$p = J_{\pm\frac{n-1}{2}}(x)/x^{\frac{n-1}{2}}, \qquad X = J_{\pm\frac{n+1}{2}}(x)x^{\frac{n+1}{2}}, \tag{35}$$

Examples.

$$n = 0, \qquad n = 2, \qquad n = -2$$

$$J_{\frac{1}{2}}(x) = \sin x/\sqrt{x}, \qquad J_{\frac{3}{2}}(x) = \sin x/x^{\frac{3}{2}} - \cos x/\sqrt{x},$$

$$J_{-\frac{1}{2}}(x) = \cos x/\sqrt{x}, \qquad J_{-\frac{3}{2}}(x) = -\sin x/\sqrt{x} - \cos x/x^{\frac{3}{2}}$$

These include the straight cylinder, the straight cone, and the purely hyperbolic horn. In the latter case we have figure 5, where $x_1$, is the bell. If the horn is closed at $x_2$ we have

$$Z_2 = \infty$$

$$Z_1 = ck^2\left\{\frac{1}{c} - \frac{k}{2\pi}i\right\} = -\frac{d}{c} = \frac{(\sin kl + kx_1\cos kl)\,\beta x_1}{\sigma_0 k \sin kl_1}$$

and if we put $\xi = kl$

$$\operatorname{ctn}\xi = \frac{\sigma_1}{lc}\xi - \frac{l}{x_1}\frac{1}{\xi}, \tag{36}$$

which may be easily discussed graphically.

On the other hand if the horn is open at $x_2$ we have

$$\tan\xi/\xi = \left(1 - \frac{\sigma x_1 x_2}{cl^3}\xi^2\right)\Big/\left(1 + \xi^2\left\{\frac{x_1x_2}{l^2} - \frac{\sigma_1 x_1}{cl^2}\right\}\right). \tag{37}$$

These formulae were confirmed experimentally by my then assistant Dr. H. K. Stimson in 1915 on a coach-horn, a trombone, and a phonograph horn, with the following results:

| | | CALCULATED | OBSERVED |
|---|---|---|---|
| For the coach-horn | Closed | 177 | 181 |
| | Open | 254 | 202 |
| For the trombone | Closed | 286 | 305 |
| | Open | 418 | 432 |
| For the phonograph | Closed | 311 | 304 |
| | Open | 329 | 415 |

These results give a fair agreement considering that we have used for the conductivity of the mouth the simple formula $c = 0.6\,R$ which is true only for cross-sections infinitesimal compared with the wave-length, whereas in the case of the wooden phonograph horn, the actual radius is nearly one-fourth of the wave-length.

A paper on the subject of the impedance of such an end will shortly appear.

In the case of an exponential section we have

$$\sigma = \sigma_0 e^{-mx}$$

$$\frac{d^2p}{dx^2} + m\frac{dp}{dx} + p = 0, \qquad \frac{d^2X}{dx^2} - m\frac{dX}{dx} + X = 0,$$

$$p = e^{-\sqrt{\frac{1}{4}-k^2}x}\{A\cos kx + B\sin kx\},$$

$$X = e^{-\sqrt{\frac{1}{4}-k^2}x}\{C\cos kx + D\sin kx\}.$$

and it is noticeable that the pressure vanishes at the same cross-section as for a straight tube.

Finally, in the case

$$\sigma = \sigma_0 e^{-mx^2}$$

we may solve the equation by means of the confluent hyper-geometric function.

It is to be noticed that in none of these cases, except the straight tube, are the different overtones harmonic. Thus, the characteristic tone of the "brass" is not due to the substance, but is entirely a matter of geometry as is shown by the heavy casting in plaster of Paris of a trombone bell used by the writer, the tone of which cannot be distinguished from that of the brass bell. I believe this phenomenon is well known.

Inasmuch as all musical instruments are composed either of resonators combined with strings, bars, plates, and horns, I feel that the above theory, while merely an approximation as to accuracy, will go far toward enabling us to complete the theory of musical instruments. Of course, the actual tones emitted by a brass instrument will depend upon the dynamics of the lips which is reserved for a future paper.

# PHENOMENA OF CELL DIVISION IN THE CAMBIUM OF ARBORESCENT GYMNOSPERMS AND THEIR CYTOLOGICAL SIGNIFICANCE

By Irving W. Bailey

Bussey Institution for Research in Applied Biology

Communicated by W. M. Wheeler, May 8, 1919

In connection with a series of investigations upon the variations in size and structure of tracheary cells in vascular plants, the writer had occasion to search for information concerning the processes of growth and cell division in the cambium of arborescent plants. He was unable to find satisfactory descriptions of these phenomena in the botanical literature, and was assured by several cytologists that comparatively little was known about them; in all probability, owing to the difficulty of sectioning the lateral meristem (cambium) of woody plants. It seemed advisable, accordingly, to initiate a special histological and cytological investigaton of this important tissue.

During 1917 and 1918, specimens of the cambium of *Pinus Strobus* L. were collected—from trees of different ages and varying environments—at frequent intervals during the growing season, April to September. Methods were perfected for fixing, sectioning and staining this material, which was compared with that secured from 14 other genera of Coniferae. The results of this investigation and of a similar study of arborescent dicotyledons will be published *in extenso* at a later date, but certain of the cytological phenomena encountered are so significant as to merit a brief preliminary description in these pages.

The cambium, although essentially an undifferentiated or 'embryonic' type of tissue, is composed of cells which are considerably elongated. In *Pinus Strobus* L., as in other arborescent gymnosperms, the cambial initials are commonly from one hundred to several hundred times as long as they are wide (radial diameter). From the point of view of Sachs' and Strasburger's theories of the 'working sphere of the nucleus' and the observations of Treub, Kallen, Buscalioni, Pirotta and Buscalioni and others upon the occurrence of many nuclei in elongated protoplasts of the higher plants, it might have been expected *a priori* that these long meristematic cells would contain more than one nucleus each.[1]

This did not prove to be the case, however, in any of the material examined by the writer. Each cell contains a single nucleus, which is centrally located and placed so that its longest axis is approximately parallel to the long axis of the cell.

During mitosis, on the other hand, the polar axis of the division figure—late prophase, metaphase, anaphase, early telophase—*does not stand*, in most cases at least, *at right angles to the long axis of the cell*.[2]

As is shown in figure 1 (*A* and *D*) the mitotic figure is placed diagonally across the cell, at an angle of from 20 to 40 degrees. That this phenomenon

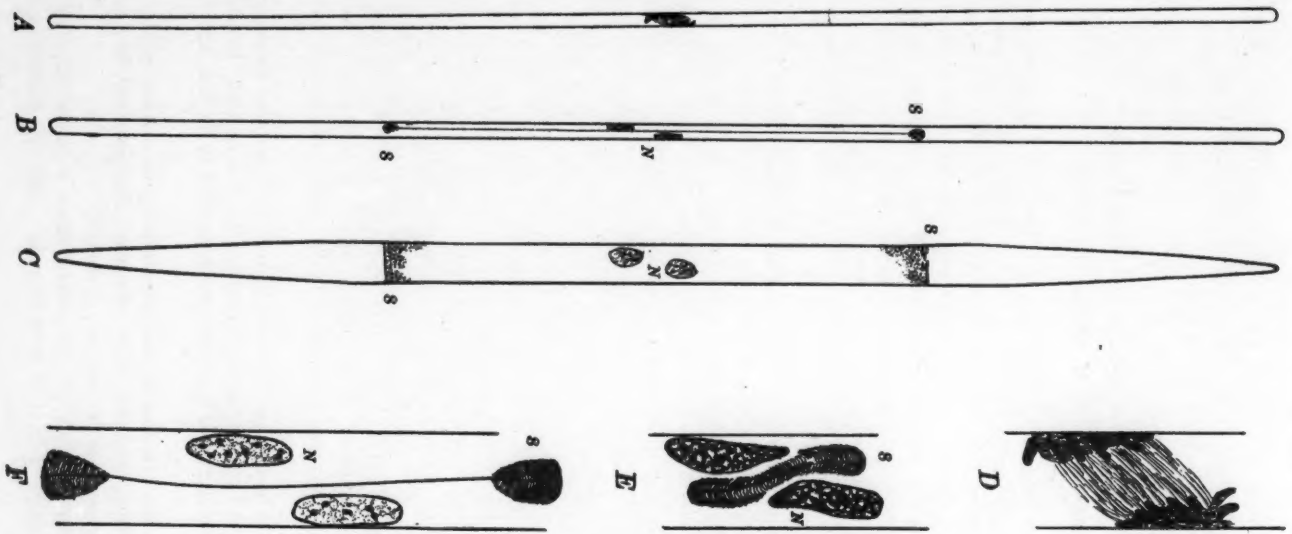



FIG. 1. PINUS STROBUS L.

*A*, Cambial initial, figured in radial and longitudinal extensions, showing oblique position of karyokinetic figure; *B*, the same, showing daughter nuclei and process of cell-plate formation; *C*, cell B, figured in tangential and longitudinal extensions; *D*, highly magnified view of karyokinetic figure shown in *A*; *E*, central portion of cambial initial, figured in radial extension, showing the beginning of cell-plate formation; *F*, the same, showing later stage of cell-plate formation; *n*, daughter nuclei; *s*, aggregations of kinoplasmic fibers, kinoplasmosomes, which are concerned in the formation of the division membrane.

is not an artifact, i.e. due to the displacement of an ordinary spindle, is indicated by the fact that the whole mitotic figure is asymmetrically developed, in conformity with its diagonal position (*D* and *E*).

Of course it is well known that the longitudinally dividing cells of the cambium form an exception to Sachs' law of 'rectangular intersection of successive division-planes' and Hertwig's modification of Sachs' hypothesis, but it has been assumed by Giesenhagen and others that, in the cambium, the karyokinetic figures lie with their polar axes perpendicular and their equatorial planes parallel to the long axes of the cells.

The formation of a cell-plate starting from one of these obliquely placed spindles is a very interesting phenomenon. The spindle becomes greatly extended laterally by the addition of peripheral fibers and gradually assumes the curved form shown in (*E*). As more peripheral fibers are successively added the remains of the central fibers disappear from about the cell-plate, leaving two separate aggregations of fibers which are connected by the first formed portion of the cell-plate (*F*). These aggregations of kinoplasmic fibers, which may be called *kinoplasmasomes*, have a very characteristic form and structure. They extend across the cell—at right angles to its longitudinal axis—from one radial wall to the other (*C*), and are located in the centre of the protoplast midway between its tangential surfaces (*F*). In sectional view (*F*), they have a somewhat wedge-shaped outline, bluntly convex in front and tapering to a point at the rear along the cell-plate. The *kinoplasmasomes* move in opposite directions towards the ends of the cell (*B* and *C*). As they move forward, the cell-plate is extended until it eventually reaches the two ends of the cell, thus dividing the protoplast into two similar portions each of which contains one of the daughter nuclei. The latter remain close together near the centre of the cell during the process of cell-plate formation. The writer has been unable to demonstrate any visible connection between the daughter nuclei and the kinoplasmasomes or their constituent fibers. Except at the beginning of cell-plate formation the daughter nuclei are in a semi-'resting condition' and are provided with a clearly defined nuclear membrane and numerous nucleoli. Not infrequently the distance traversed by the kinoplasmasomes, in passing from the vicinity of the daughter nuclei to the ends of the protoplast, may be from one to several millimeters.

This type of cell division, in which the process of cell-plate formation is so greatly extended—both as regards space and time—and so clearly dissociated from the usual phenomena of karyokinesis, promises, upon further analysis, to be of some significance in the discussion of the dynamics of cell division.

[1] Schacht and Russow claimed to have seen several nuclei in the cambial cells of Pinus.

[2] The writer is dealing with the normal longitudinal divisions of the cambial initials.

# ON THE POSSIBLE FORM OF THE EQUATION OF STATE OF POWDER GASES

By A. G. Webster

Ballistic Institute, Clark University, Worcester, Massachusetts*

Communicated May 14, 1919

It has been customary for ballisticians to make use of the equation proposed by Clausius,

$$\left\{p + \frac{a}{T(v+\beta)^2}\right\}(v-\alpha) = RT, \tag{1}$$

in the simplified form, suitable for the high temperatures concerned,

$$p\,(v - \alpha) = RT. \tag{2}$$

At the same time it is customary to make use of the experimental results of Mallard and le Chatelier and of Berthelot and Vieille on the specific heats which state that $C_v$ is a linear increasing function of the temperature. While apparently no experiments have been made on $C_p$ it is assumed that the difference of the specific heats is constant, as in the case of an ideal gas.

It has occurred to me to examine the question of the most general form possible for the equation of state that shall permit of variability of the specific heats, but maintain the constancy of their difference. This question does not appear to have been treated,

By an application of the two laws of thermodynamics we obtain the well-known equation

$$(C_p - C_v)\frac{\partial T}{\partial p}\frac{\partial T}{\partial v} = T. \tag{3}$$

If we use the usual letters for differential equations, putting $x$ for $v$, $y$ for $p$, $z$ for $T$ divided by $C_p - C_v$ supposed constant, and as usual $p$ for $\partial z/\partial x$, $q$ for $\partial z/\partial y$ we have the very simple partial differential equation,

$$F = p\,q - z = 0. \tag{4}$$

This may be very simply integrated by Cauchy's method, which consists in integrating the system

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{Pp + Qq} = -\frac{dp}{X + pZ} = -\frac{dq}{Y + qZ} = \frac{du}{q},$$

where the capital letters represent the derivatives of $F$ with respect to the corresponding small letters, and $u$ is an extraneous parameter. Having found

* Contribution from the Ballistic Institute, Clark University, No. 5.

five integrals, with five arbitrary constants $x_0, y_0, z_0, p_0, q_0$ we make the latter functions of a second parameter $v$ satisfying the equations

$$p_0 q_0 - z_0 = 0, \quad \frac{\partial z_0}{\partial v} = p_0 \frac{\partial x_0}{\partial v} + q_0 \frac{\partial y_0}{\partial v}. \tag{5}$$

We easily obtain the five integrals,

$$x - x_0 = u, \quad y - y_0 = \frac{p_0}{q_0} u, \quad \frac{p}{p_0} = \frac{q}{q_0}, \quad q - q_0 = u,$$

$$z - z_0 = 2 p_0 u + \frac{p_0}{q_0} u^2, \text{ with } p_0 = \frac{z_0}{q_0}.$$

Instead of adopting Cauchy's form for the introduction of the arbitrary function, we will attempt to pass the integral surface through the plane $z_0 = const.$, representing an isothermal. We put

$$\begin{aligned} &x_0 = v, \quad y_0 = \varphi(v), \quad p_0 + q_0 \varphi'(v) = 0, \quad \frac{z_0}{q_0^2} = -\varphi'(v), \\ &y = \varphi(v) - u\varphi'(v), \\ &z = z_0 \pm 2\sqrt{-z_0 \varphi'(v)}\, u - \varphi'(v) u^2, \\ &x = u + v. \end{aligned} \tag{6}$$

If we adopt the Clausius equation for the form of one particular isothermal, we may put

$$\begin{aligned} \varphi(v) &= \frac{Rz_0}{v - \alpha} - \frac{a}{z_0 (v + \beta)^2}, \\ \varphi'(v) &= -\frac{Rz_0}{(v - \alpha)^2} + \frac{2a}{z_0 (v+\beta)^3}. \end{aligned} \tag{7}$$

We thus obtain finally

$$\begin{aligned} &x = u + v, \\ &y = \frac{Rz_0}{v - \alpha} - \frac{a}{z_0 (v + \beta)^2} + u \left\{ \frac{Rz_0}{(v - \alpha)^2} - \frac{2a}{2_0 (v + \beta)^3} \right\} \\ &z = z_0 \pm 2u \sqrt{z_0 \left( \frac{Rz_0}{(v - \alpha)^2} - \frac{2a}{z_0 (v + \beta)^2} \right)} + u^2 \left\{ \frac{Rz_0}{(v - \alpha)^2} - \frac{2a}{z_0 (v + \beta)^3} \right\} \end{aligned} \tag{8}$$

so that we have the parametric equation of the surface. It may be noted that putting $u = 0$, $z_0 = T$ we fall back on the ordinary Clausius equation (1) as a particular case, with (2) and the ideal gas equations as still more particular.

In order to obtain the expression for the energy for such a gas, we make use of the equation

$$U = \int \left[ \left( T \frac{\partial p}{\partial T} - p \right) dv + C_v dT \right], \tag{9}$$

in which we have now to put

$$\frac{\partial p}{\partial T} = \frac{\partial(p, v)}{\partial(x, y)} \Big/ \frac{\partial(T, v)}{\partial(x, y)}$$

We have now to make use of equations (6) in which, replacing the usual thermal notation, and now using $x$ and $y$ for the arbitrary parameters,

$$\begin{aligned} T &= T_0 \pm 2x \sqrt{-T_0\varphi'(y)} - x^2\varphi'(y), \\ p &= \varphi(y) - x\varphi'(y), \\ v &= x + y, \end{aligned} \tag{10}$$

$$\frac{\partial p}{\partial x} = -\varphi'(y), \quad \frac{\partial p}{\partial y} = \varphi'(y) - x\varphi''(y), \quad \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = 1,$$

$$\frac{\partial T}{\partial x} = \pm 2\sqrt{-T_0\varphi'(y)} - 2x\varphi'(y),$$

$$\frac{\partial T}{\partial y} = \varphi''(y) \sqrt{-\frac{T_0}{\varphi'(y)}} - x^2\varphi''(y),$$

so that finally

$$U = \int \left[ \left( T_0 \pm 2x \sqrt{-T_0\varphi'(y)} - x^2\varphi'(y) \right) \left\{ \frac{-2\varphi('y) + x\varphi''(y)}{2\sqrt{-T_0\varphi'(y)} - 2x\varphi'(y) - \varphi''(y)\sqrt{-\frac{T_0}{\varphi'(y)}}} \right\} \times \left\{ \frac{\partial v}{\partial x} dx + \frac{\partial v}{dy} dy \right\} + C_v \left\{ \frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial y} dy \right\} \right] \tag{11}$$

I have also integrated the equation for the case that the difference of the specific heats is a linear function of the temperature, but this seems not necessary in the light of present experimental data.

# THE RELATIVE ADSORPTION OF MIXTURES OF OXYGEN AND NITROGEN IN COCOANUT SHELL CHARCOAL[1]

By Harvey B. Lemon and Kathryn Blodgett

Ryerson Physical Laboratory, University of Chicago

Communicated by A. A. Michelson, May 19, 1919

It is a well known fact that gases are adsorbed in charcoal with rates and in total amounts that vary in a manner closely related to the boiling temperatures of the gases. A mixture of gases may accordingly have its proportions entirely altered by adsorption. This is the principle of the method of Gehloff for the isolation of atmospheric neon.[2] Dewar has mentioned it as a convenient means of extracting a high percentage of oxygen from the air.[3] A quantitative knowledge of the manner in which the presence of one gas to saturation affects the adsorption of another is of great importance since these are the conditions of use under which charcoal has sprung into prominence in modern warfare.

The experiments herein described deal with the relative adsorption of mixtures of oxygen and nitrogen in varying proportions by a highly activated charcoal prepared in the manner described in a previous report by one of the writers. Relatively large amounts of charcoal are employed with respect to the quantity of gas used so that saturation is in all cases reached in the course of thirty minutes or so. The charcoal which weighed 6.5 grams when saturated with dry air at 20° and 750 mm. pressure was contained in Pyrex glass bulbs which could be outgassed by a diffusion pump. Outgassing was for four and a quarter hours at 582°C. After outgassing the tubes were immersed in liquid air of definite age and temperature. While immersed they were exposed to the given gas mixture contained in a constant volume of 975 cc. The initial pressure of the mixture was 73.95 cm. and observations were taken of it at intervals while the adsorption was going on and until it had ceased to fall. A barometer and McLeod gauge formed a part of the above mentioned volume for this purpose.

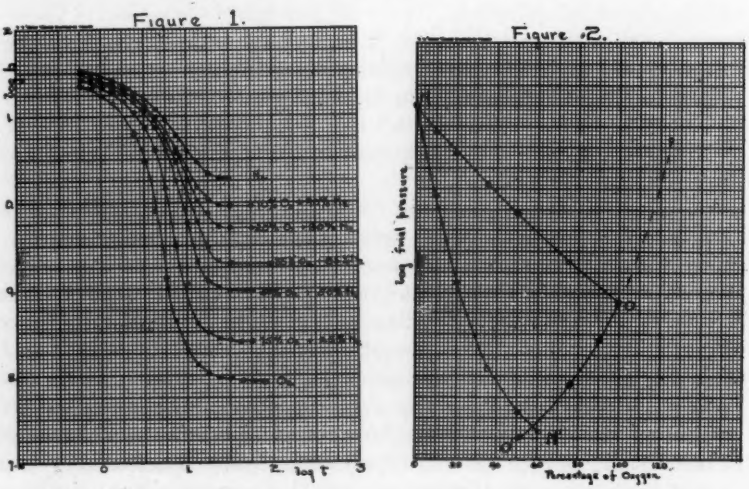
In figure 1 is given the data in graphical form where log $p$ in cm. is plotted against log $t$ in minutes after the adsorption began. The same sample always is included in this record, two others were used as controls. It is noted that the logarithm of the pressure reached by saturation is almost in a linear relation with the percentage of oxygen in the mixture.

This is shown in figure 2 as the line $NO$. Here log final pressure is plotted against percentage of oxygen.

The other lines of this diagram $NN'$ and $OO'$ show the final pressures of amounts of pure nitrogen and pure oxygen equal to those existing in the mixture at corresponding values of abscissa but adsorbed separately.

It is important to note that in no case does the final pressure of a mixture fall as low as the sum of the final pressures of the two components when adsorbed separately. In other words, each component in the mixture hinders the adsorption of the other. This is not in agreement with an earlier paper by Bergter[4] who concludes that "in the range of pressure investigated the ability of charcoal to adsorb nitrogen must be increased by the presence of oxygen." Bergter's pressure range was entirely below 1 mm. and the discrepancy may be due to this difference of experimental conditions.

In the course of the work it became of importance to know if the final adsorption would be the same if a given amount of pure gas were adsorbed all at once as it would if half were adsorbed at one time and then the second half




admitted later. It was found that adsorption is much more complete in the second case, the quantitative pressures being as follows:

| | *Final pressure* | *Log p* |
|---|---|---|
| a—Oxygen 100 per cent | 0.00797 | 7.902–10 |
| b—Oxygen 50 per cent, 50 per cent | 0.00604 | 7.781–10 |
| a—Nitrogen 100 per cent | 1.86 | 0.270 |
| b—Nitrogen 50 per cent, 50 per cent | 0.649 | 9.812–10 |

In adsorbing equal parts oxygen and nitrogen it seems to be immaterial which is admitted first, the final pressure being within experimental limits the same.

| | | |
|---|---|---|
| Oxygen 50 per cent, Nitrogen 50 per cent | 0.0451 | 8.654–10 |
| Nitrogen 50 per cent, Oxygen 50 per cent | 0.0481 | 8.682–10 |
| Oxygen and nitrogen mixed each 50 per cent | 0.0924 | 8.966–10 |

If a mixture of the two gases is adsorbed then as above the final pressure reached is considerably higher.

[1] This article is published with the approval of Major General William L. Siebert, Director Chemical Warfare Service, U. S. A.

[2] Gehloff, *Leipzig, Physik, Zs.*, 7, 1913, (838).

[3] Dewar, *London, Proc. Roy. Soc.*, 74, 1904, (122).

[4] Bergter, *Leipzig, Ann. Physik*, 37, 1912, (606).

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## *VARIATIONS, DUE TO HEAT TREATMENT, IN THE RATE OF ADSORPTION OF AIR BY COCOANUT SHELL CHARCOAL*[1]

By Harvey B. Lemon

Ryerson Physical Laboratory, University of Chicago

Communicated by A. A. Michelson, May 19, 1919

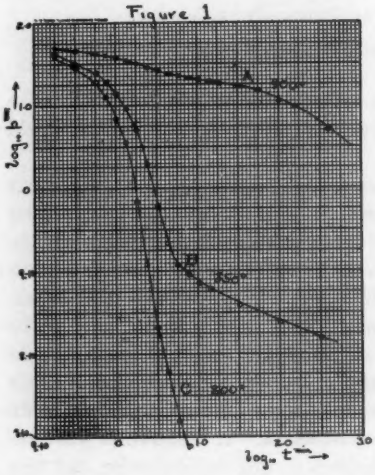
This paper is an abstract of results of experiments most of which were performed prior to January 1918, but the publication of which has been withheld during the war.[1]

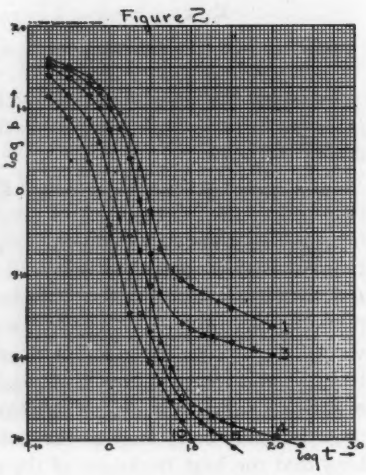
If charcoal contained in a bulb of glass or iron is heated to 600°C for about four hours and the gases which are freed pumped off into the low vacuum furnished by a mercury diffusion pump,[2] it is a well known fact that when the charcoal is then subsequently cooled to the temperature of liquid air it possesses a tremendous capacity for adsorbing gas, and will adsorb it at an extremely rapid rate. This adsorption power, however, has been found to be very different with different specimens of charcoal made from the same material, in this case cocoanut shell, and also to be very different when a single sample is used repeatedly. The magnitude of these differences is of no mean order but may be as large as the ratio of 10,000 : 1. It is shown in what follows that the heat treatment of the specimen during carbonization and also during successive 'outgassings' is a decisive factor in the control of the efficiency of the material as an adsorbent.

The experimental method was one of extreme simplicity. The shell was carbonized in an enclosed electric furnace having a vent for escape of gases and vapors. The temperature was indicated on a Leeds Northrup potential point resistance thermometer. The charcoal was then ground up to particles of from 1 to 3 mm. diameter and cleansed from all smaller fragments and dust. A definite weight (25.7 grams in most of the experiments, determined in dry air with which the charcoal was saturated) was sealed up in a tube of iron, quartz or Pyrex glass depending on the temperatures to be subsequently used. From this tube cocks communicated, (1) to the diffusion pump, and (2) to a fixed volume that could be filled with dry air at any desired pressure. This fixed volume included a McLeod gauge and a mercury barometer column so that the pressure in it could be read to within a few per cent over a range

of from 100 cm. to 0.00001 cm. A small Geissler tube was in communication with the charcoal bulb for a rapid means of observing the character of the gas content and, qualitatively, the pressure. After outgassing through cock, *1*, this was closed and cock, *2*, opened. The charcoal was thus exposed to a constant volume of 873 cc. initially filled with air at about 90 cm. pressure. This initial pressure was varied slightly as the room temperature varied so that the enclosed mass of air was constant, i.e. 1.2 grams. The air was immediately adsorbed and the rate of fall of the pressure observed as long as it was appreciable.

Results are expressed graphically by plotting the logarithm of the pressure in cm. against the logarithm of the time in minutes elapsing after the cock, *2*, was opened. Figure 1 shows curves for three different samples under




identical conditions. *A* was carbonized at 900°C., *B* at 850°C. and *C* at 800°C. They were all outgassed simultaneously for 6 hours at 425°C. and tested in rapid succession on the same apparatus. The enormous difference in rate is obvious in view of the logarithmic scales. The initial pressure of 90 cm. is reduced in 10 minutes to 20 cm., 0.71 cm. and 0.0003 cm. for *A*, *B* and *C* respectively.

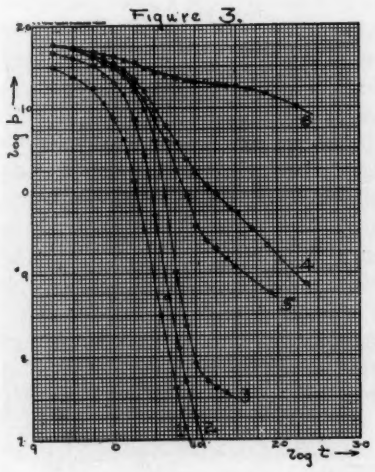
Figure 2 shows the results of successive 'runs' on the same specimen. This specimen was carbonized at 850°C. and the outgassings were all identical at 600° for 4½ hrs. It shows a systematic increase in rate after each run and subsequent outgassing becoming equal after 10 consecutive runs to the former sample shown as *C* in figure 1.

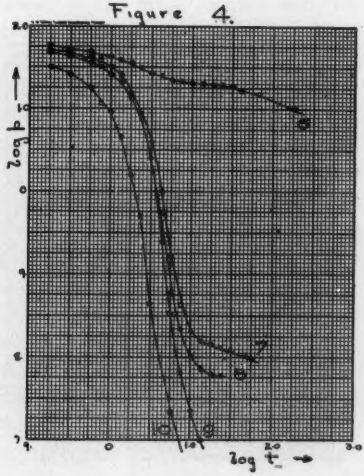
Figure 3 shows an effect which was controlled so as to be the converse of that illustrated in figure 2. It depicts also successive runs on a single speci-

men, the carbonization of which was for 3 hours at 670°C. The first run shows an extremely active adsorption. Successive outgassings were not now identical but were as indicated below:

| | |
|---|---|
| First | 4.5 hours at 633°C. |
| Second | 0.5 hours at 800°C. |
| Third | 0.5 hours at 850°C. |
| Fourth | 0.5 hours at 875°C. |
| Fifth | 4.0 hours at 640°C. |
| Sixth | 0.7 hours at 905°C. |

The first four runs show a cumulative loss of activity as result of high outgassing temperature. In the fifth the activity is in part restored by prolonged outgassing at the lower temperature and in the sixth it is again almost




totally destroyed. Quartz tubes are unsuitable for use for the high temperature work because of their devitrification by the hot carbon vapor.

Figure 4, repeating the sixth run shown in the preceding, carried the same sample back to high activity in four subsequent outgassings which were as follows:

| | |
|---|---|
| Seventh | 22.0 hours at 650°C. |
| Eighth | 22.0 hours at 500°C. |
| Ninth | 44.0 hours at 650°C. |
| Tenth | 1.0 hours at 840°C. |

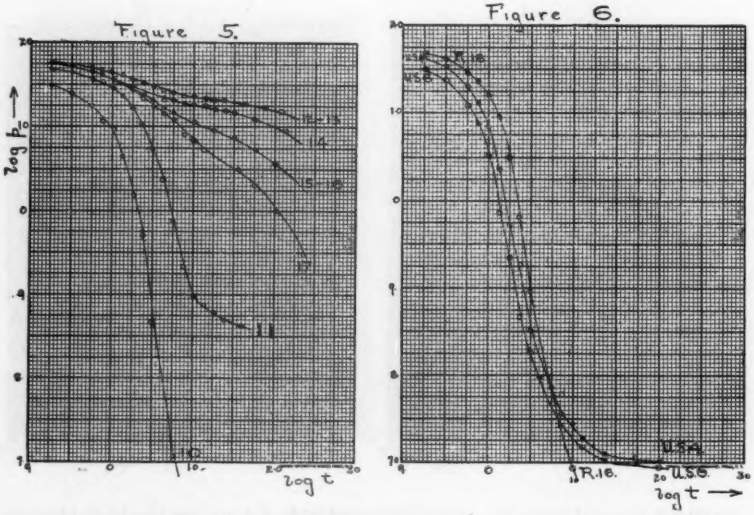
A sluggishness of behavior is apparent after this much use as indicated in the long times necessary to produce a change in quality and also in the failure to respond as before to a temperature higher than 800.

Figure 5 shows continuation of the attempt for the second time to destroy the activity. Repeating curve 10 of the preceding and outgassing

Eleventh.....0.5 hours at 950°C.
Twelfth.....1.5 hours at 985°C.

the activity is again reduced. Recovery is now as before sluggish and was not carried to completion.

Thirteenth.....3.0 hours at 650°C.
Fourteenth.....8.0 hours at 625°C.
Fifteenth.....12.0 hours at 645°C.
Sixteenth.....1.5 hours at 650°C.
Seventeenth.....14.0 hours at 650°C.




The characteristic form of the family of curves shown is undoubtedly of generic quality. These curves have been reproduced on a wide variety of samples of cocoanut shell charcoal many times; and when the results are assembled exhibit almost every degree of gradation from a central, closely linear (after the first minute) relationship. Similar experiments on pure gases rather than air are in progress in the hope of simplifying this family of curves somewhat so as to obtain an interpretation expressible in analytic form.

Two hypotheses have been advanced for these phenomena. One ascribes the changes in the sense of increased activity to the gradual distillation out of the material of heavy nonvolatile hydrocarbons, many of which in the form of tars and gums come off during the initial carbonization. The other is based on the conception advanced by Miss Ida Homfray[3] that these phe-

nomena are to be regarded as saturated solutions of a rigid phase, carbon, in a fluid phase, gas. It suggests that repeated solution and subsequent evaporation, here taking the form of adsorption at low temperatures and outgassing at high, produces a gradual modification of the character of the carbon with respect to its fineness of division, something similar to precipitation occurring.

Loss of activity seems difficult to account for on the first hypothesis. It is to be noted that permanent loss of activity can always be produced on any sample by heating to 1200°C. This is usually ascribed to a partial destruction of porosity and has been observed before. Attempts to extract heavy hydrocarbons by the use of the lightest liquid solvents, ligroin alcohol and acetone were inconclusive. Considerable amounts of tarry material were removed, but not by the solvents directly. They distilled out during outgassing after treatment with the solvent. This treatment in all cases causes temporary loss of activity which is renewed in the usual manner with repeated use after low temperature outgassings. Experiments by others however[4] which have shown that activation is possible by other methods than the ones here outlined seems to favor the hydrocarbon hypothesis.

Those other methods for activation of field material for the adsorption of complex vapors have been used in conjunction with the above described process. A comparison of material activated by use and low temperature outgassings alone as herein described, with the most highly activated charcoals produced elsewhere under the conditions of these experiments is given in figure 6. The crossing of the curves may be very significant. The laboratory charcoal, R. 16, is less active initially but ultimately runs to lower values than the field material U. S. 4 and U. S. 6. The differences at either end are not large in comparison with the range of the phenomena discussed above.

Experiments are in progress on saturation values for adsorptions of mixtures of varied proportions. An hypothesis originally advanced by McBain[5] that there is a distinction to be made between surface condensation and interior diffusion is also being subjected to experimental scrutiny with modern materials now at our disposal. Both of these lines of work it is hoped will shed light on the mechanism of the process.

A more detailed account of this work will shortly appear in the Physical Review as a series of papers under the general title of Studies in Charcoal Adsorption.

[1] This article is published with the approval of Major General William L. Siebert, Director Chemical Warfare Service, U. S. A.

[2] Shrader, *Ithaca, N. Y. Physic. Rev.*, 12, 1918, (70).

[3] Homfray, *Liepzig, Zs. Phys. Chem.*, 74, 1910, (139).

[4] Dorsey, *Easton, Pa., J. Ind. Eng. Chem.*, 11, 1919, (284).

[5] McBain, *London, Phil. Mag.*, 18, 1909, (916).

# REAL HYPERSURFACES CONTAINED IN ABELIAN VARIETIES

By S. Lefschetz

Department of Mathematics, University of Kansas

Communicated by E. H. Moore, April 29, 1919

1. In a recent note of these Proceedings (April, 1919), I showed that an abelian variety of genus $p$ and rank one, $V_p$, is birationally transformable into a real one if and only if it possesses $2p$ independent linear cycles $\gamma_1, \gamma_2, \ldots \gamma_{2p}$, with respect to which $p$ integrals of the first kind have a period matrix of type $\Omega = \| \omega_{h,1}, \ldots \omega_{h,p}; i\omega_{h,p+1}, \ldots i\omega_{h,2p} \|$ ; $(h = 1, 2, \ldots p)$, the $(\omega)$'s being real. I propose now to investigate the number $\rho'$ of algebraically distinct real hypersurfaces which $V_p$, if real, may have. This number $\rho' \leqq \rho$, Picard number of $V_p$, may also be defined as the maximum number of real hypersurfaces which cannot be logarithmic singularities of a simple integral of the third kind.

2. In a general way $V_p$ be an abelian variety of rank one, real or not, with the independent linear cycles $\gamma_1, \gamma_2, \ldots \gamma_{2p}$. By associating $\gamma_\mu$ with $\gamma_\nu$ we obtain a superficial cycle $(\mu, \nu)$ and any other depends upon those of this type. In particular denoting by $(A^{p-1})$ the two dimensional cycle formed by $A^{p-1}$, curve of intersection of $p-1$ algebraic hypersurfaces of the same continuous system as a given one $A$, we have

$$m\,(A^{p-1}) \sim \sum_1^{2p} {}_{\mu,\nu}\; m_{\mu,\nu}\,(\mu, \nu), \quad (m_{\mu,\nu} \text{ integer} = -m_{\nu,\mu}). \tag{1}$$

It may be shown that if no integral of the first kind is constant on $A$ the alternate form

$$\Sigma\, m_{\mu,\nu}\, x_\mu\, y_\nu \tag{2}$$

is a principal form of $\Omega$ as defined by Scorza (Palermo Rendic., 1916), and conversely to a principal form (2) corresponds an algebraic hypersurface $A$. Moreover to algebraically distinct hyersurfaces correspond linearly independent principal forms from which follows at once $\rho = 1 + k$, where $k$ is Scorza's index of singularity for $\Omega$.

3. Let us now assume $V_p$ real. A real hypersurface $A$ of $V_p$ is transformed into itself by $T$, transformation of the variety which permutes its pairs of conjugate points and this property is characteristic for $A$. It may be shown that there are real curves $A^{p-1}$,—let the one of No. 2 be one of them, and $\alpha$ its real line (locus of its real points). A small oriented circuit tangent to $\alpha$ in $(A^{p-1})$ is transformed by $T$ into one of opposite orientation, for in the neighborhood of $\alpha$, $T$ behaves like an ordinary plane symmetry. It follows that $T$ transforms the superficial cycle $(A^{p-1})$ into its opposite. Taking into account the fact that this cycle is a two sided manifold and also

the effect of $T$ upon the linear cycles $\gamma_\mu$ of No. 1, we find at once that all the $m$'s not of the type $m'_{\mu,p+\nu}$, $(\mu, \nu \leqq p)$ are equal to zero, hence $\rho'$ *is equal to the number of independent forms of type.*

$$\sum_1^p{}_{\mu,\nu}\ m'_{\mu,p+\nu}\ (x_\mu y_{p+\nu} - x_{p+\nu} y_\mu) \tag{3}$$

*which belong to* $\Omega$.

If $V_p$ is pure $\rho' \leqq p$, for otherwise $\Omega$ would possess a degenerate form (3). This is to be contrasted with Scorza's result $1 + k \leqq 2p - 1$, or $\rho \leqq 2p - 1$ if $\Omega$ is pure.

4. Assuming $\rho' = 2$ let $L$, $L'$, be the matrices formed by the determinants of two forms (3). They are both of type

$$\left\| \begin{array}{c|c} 0 & \Delta \\ \hline \Delta' & 0 \end{array} \right\|$$

where each square represents a matrix with $p$ rows and columns, the matrices in the main diagonal having only zeroes for terms. As $L^{-1} L'$ is of the form

$$\left\| \begin{array}{c|c} \Delta & 0 \\ \hline 0 & \Delta' \end{array} \right\| = \| a_{\mu\nu} \|, \ (\mu, \nu = 1, 2, \ldots 2p;\ a_{\mu, p+\nu} = a_{p+\mu,\nu} = 0),$$

$V_p$ has a complex multiplication defined by

$$\sum_1^p{}_k \lambda_{j\mu}\omega_{k\mu} = \sum_1^p{}_\nu a_{\mu\nu}\omega_{j\nu};\quad \sum_1^p{}_j \lambda_{j,k}\,\omega_{k,p+\mu} = \sum_1^p{}_\nu a_{p+\mu,p+\nu}\omega_{j,p+\nu}$$
$$(j, \mu = 1, 2, \ldots p),$$

the $(\lambda)$'s being necessarily real as they can be replaced by their conjugates. Finally the characteristic equation of this complex multiplication

$$\| a_{\mu\nu} - \epsilon_{\mu\nu}\, x \| = 0,\ (\epsilon_{\mu\nu} = 0,\ \mu\ \nu;\ \epsilon_{\mu\mu} = 1)$$

is necessarily reducible and a perfect square if $V_p$ is pure.

5. Let us examine the case of a real hyperelliptic surface of rank one. The number $\rho'$ has then the value 1 or 2, if the surface is pure not elliptic. A fundamental period matrix corresponding to linear cycles forming a minimum base may be reduced to the form

$$\left\| \begin{array}{cccc} 1, & 0, & \dfrac{m}{2} + ia, & \dfrac{n}{2} + ib \\ 0, & 1/\delta, & \dfrac{n}{2} + ib, & \dfrac{r}{2} + ic \end{array} \right\|,$$

where $m$, $n$, $r$, $\delta$, are positive integers and $ac - b^2 > 0$. If $\gamma'_1, \gamma'_2, \ldots \gamma'_{2p}$, are the corresponding linear cycles those of No. 1 are given by

$$\gamma_1 = \gamma'_1,\ \gamma_2 = \gamma'_2,\ \gamma_3 = 2\gamma'_3 - m\gamma'_1 - n\delta\gamma'_2,\ \gamma_4 = 2\gamma'_4 - n\gamma'_1 - r\delta\gamma'_2,$$

and in general $\rho = \rho' = 1$, unless there is a singular relation as defined by G. Humbert. If the surface is not elliptic this relation can only be of type

$$\lambda a + \mu b + \gamma \delta c = 0, \ (\lambda, \mu, \nu, \text{integers}) \tag{3}$$

and there can only be one such relation. In this case $\rho = \rho' = 2$, and the condition of existence becomes now, assuming as we may, $\nu > 0$,

$$\lambda a^2 + \mu ab - \nu \delta b^2 > 0$$

which assures us of the effective existence of the surface. If there are two singular relations such as (5) the surface is elliptic and $\rho = \rho' = 3$.

In addition to (5) there may be in the non-elliptic case as well as in the other a singular relation independent of (5) and reducible to the form

$$\lambda (b^2 - ac) + \mu = 0, \ (\lambda, \mu, \text{positive integers})$$

and then $\rho - \rho' = 1$, both cases being realizable. Thus there are six distinct types of real hyperelliptic surfaces for which $\rho, \rho'$ have the values: (1, 1), (1, 2), (2, 2), (2, 3), (3, 3), (3, 4), the last three corresponding to elliptic cases.

# NATIONAL RESEARCH COUNCIL

## EXTRACTS FROM THE MINUTES OF THE MEETING OF THE EXECUTIVE BOARD

AT THE NATIONAL RESEARCH COUNCIL BUILDING, APRIL 15, 1919, AT 9.30 A.M.

Present: Messrs. Bancroft, Clevenger, Cross, Dunn, Flinn, Hale, Howe, Hussey, Johnston, Leuschner, Merriam, Millikan, Noyes, Walcott, Washburn and Yerkes. Mr. Hale in the chair.

The minutes of the meeting of the Executive Board, March 11 and of the meetings of the Interim Committee, March 18, 25, April 1, and April 8, were approved. The actions taken by the Interim Committee are included in the record of these minutes.

At these various meetings reports were presented by the Chairman of the Council and by the Chairmen of the Divisions of Science and Technology on the progress of the nominations from scientific and technical societies for representatives to serve on the Divisions of Science and Technology of the Council, on the organization meetings of various Divisions and on the election of officers, executive committees, and sub-committees of the Divisions. In accordance with action taken by the Executive Board, each divisional organization will be printed in the PROCEEDINGS after its completion and subsequent approval by the Executive Board, except that sub-committees of Divisions will be reported in the Minutes of the meeting at which they are approved. The membership and officers of the Divisions of Physical Sciences, of Engineering, of Chemistry and Chemical Technology, of Geology and Geography, and of Biology and Agriculture are announced in the present number.

*Moved:* That the first meeting of each of the permanent Divisions, at which the Division is organized and elects its officers and executive committee, be recognized as the annual meeting of the Division for this year. (*Adopted.*)

*Moved:* That all appointments made hereafter shall be considered to terminate on the last day of June of the appropriate year. (*Adopted.*)

*Moved:* That the question as to when the present membership of the Council ceases be referred to the Committee on Orzanization with the request that they formulate and circulate resolutions regarding this matter before the next meeting of the Interim Committee. (*Adopted.*)

*Moved:* That the Secretary be requested to bring to the attention of the Divisions the provisions of Section 6, Article V, for fixing by lot the term of office of the new members of the Divisions. (*Adopted.*)

*Moved:* That the Division of Physical Sciences be authorized to appoint a special committee to consider the advisability of enlarging the functions of the International Bureau of Weights and Measures and to prepare suitable recommendations in this respect.

*Moved:* That an additional allotment of $1,000 be made to the general maintenance fund of the Engineering Division; and that the transportation expenses of the members of the Division attending the first meeting, be met by the Council. (*Adopted.*)

*Moved:* That an appropriation of $500 be made to the Executive Board to cover the expense of preparation of an annotated analytical bibliography, according to the scheme proposed by Mr. Fulcher, of the literature dealing with the effects of over-strain and blue heat on steel. (*Adopted.*)

*Moved:* That in view of Mr. Hale's desire to be relieved of his duties as Chairman of the Council at an early date Mr. J. C. Merriam be elected Acting Chairman as from March 15. (*Adopted.*)

*Moved:* That there be constituted an Interim Committee of the Executive Board which shall include in its membership the Chairman, Secretary, and Treasurer of the Council and the Chairman (or in his absence the Vice-Chairman) of each of the Divisions of the Council; that the Chairman of the Council shall be Chairman of the Interim Committee; and that this Interim Committee shall act for the Executive Board between meetings of the latter. (*Adopted.*)

*Moved:* That reports of the conferences of members of the Council with research representatives of Middle West and Pacific Coast institutions, held at Chicago and at Berkeley, California, on February 1st and March 8th, respectively, be incorporated in the minutes of the Council. (*Adopted.*)

*Moved:* That an address entitled 'Industrial Research' by Dr. Frank B. Jewett, which was presented before the Royal Canadian Institute, Toronto, February 8, 1919, be published as a bulletin of the National Research Council. (*Adopted.*)

The Chairman of the Council presented a communication from the Secretary of the General Education Board, informing the Council that the Board had made an appropriation of $25,000 for the purpose of a study to be conducted under the auspices of the National Research Council by Mr. Robert M. Yerkes and Mr. Lewis M. Terman for the purpose of developing intelligence tests for the mental rating of school children, payments to be made to the National Academy of Sciences.

*Moved:* That the National Research Council express its appreciation of the action of the General Education Board in appropriating a sum of $25,000 for the purpose of developing intelligence tests for the mental rating of school children, and extend a vote of thanks for this gift. (*Adopted.*)

*Moved:* That the sum of $450, or so much thereof as may be necessary, be appropriated from the funds available for the Engineering Division to cover the transportation expenses of certain members of the Committee on the Use of Tellurium and Selenium attending a meeting of the Committee to be held in Buffalo. (*Adopted.*)

Mr. Washburn presented a brief report of the initial meeting of the new Division of Chemistry and Chemical Technology held in Washington, March 21 and 22.

*Moved:* That the American Ceramic Society be invited to form with the Division a joint Committee on Ceramic Research. (*Adopted.*)

*Moved:* That the Committee on Synthetic Drugs be continued; and that the Committee on Explosives Investigations be continued temporarily. (*Adopted.*)

*Moved:* That the Council authorize the sending of a delegate to a meeting called by the Société de Chimie Industrielle to be held in Paris April 14, at which certain international chemical questions will be discussed and that the selection of this delegate be left to the Chairman and Chairman-elect of the Division, and the Chairman of the National Research Council. (*Adopted.*)

*Moved:* That Mr. Leuschner be elected Acting Secretary of the National Research Council from April 16 to June 30, 1919. (*Adopted.*)

*Moved:* That the Chairman appoint a Committee to consider all matters relating to organization of administration. (*Adopted.*)

Appointed: Messrs. Cross (Chairman), Clevenger, Hussey, Leuschner, Yerkes.

*Moved:* That the present Budget Committee be discharged and that the Chairman appoint a committee to consider the budget for the ensuing year. (*Adopted.*)

Appointed: Messrs. Leuschner (Chairman), Cross and Clevenger.

*Moved:* That the transportation expenses of the members of the Division of Biology and Agriculture attending the first meeting of the Division be met by the Council. (*Adopted.*)

*Moved:* That the President of the National Academy of Sciences be requested to appoint as members of the Research Council the representatives of societies and others nominated to membership in the Divisions of Engineering, Chemistry, and Physical Sciences, for such terms as have been or may be determined by lot in the Divisional meetings. (*Adopted.*)

The Chairman of the Division of Engineering recommended the approval of the following Committees:

*Nominating Committee.*—C. A. Adams (Chairman), E. P. Hyde, and Ambrose Swasey.

*Executive Committee.*—C. A. Adams, D. S. Jacobus, E. G. Spilsbury, and the Chairman and Vice-Chairman of the Division.

*Publication Committee.*—A. M. Greene (Chairman), E. P. Hyde, and E. G. Spilsbury.

*Finance Committee.*—W. R. Walker, (Chairman), Gano Dunn, C. F. Rand, and A. A. Stevenson.

*Heat Treatment of Carbon Steel.*—Henry N. Howe (Chairman), R. M. Boylston, and Albert Sauveur.

*Pulverizing.*—G. H. Clevenger (Chairman).

*Committee on Insulation.*—F. B. Jewett (Chairman), C. A. Adams, Leroy Clark, Wallace S. Clark, W. A. Delmar, F. M. Farmer, A. E. Kennerly, F. W. Peek, H. J. Shanklin, C. E Skinner, J. B. Whitehead.

*Moved:* That the Committees as recommended by the Engineering Division be authorized and that the appointment of the members as given above be approved. (*Adopted.*)

Mr. Clevenger presented a request of the American Welding Society that the National Research Council designate a representative for appointment as Director in the American Welding Society.

*Moved:* That in accordance with a request received from the American Welding Society the Division of Engineering be authorized to appoint a member of the Division as director in the American Bureau of Welding with full powers. (*Adopted.*)

The Acting Chairman, Mr. Merriam, presented a communication from Mr. W. A. Averill of the Central Bureau of Planning and Statistics requesting that a weekly report be presented on Friday of each week on the work of the National Research Council.

*Moved:* That in accordance with a request received from the Central Bureau of Planning and Statistics weekly reports of activities of the Research Council be transmitted to the Central Bureau of Planning and Statistics. (*Adopted.*)

The Acting Chairman, Mr. Merriam, announced the appointment on April 4, of the following committees and requested that these appointments be approved:

Committee on Organization *of the Division of Industrial Relations.*—Henry M. Howe, (Chairman), W. D. Bancroft, G. H. Clevenger, and E. F. Nichols.

Committee on Organization *of the Government Division.*—R. M. Yerkes (Chairman), W. D. Bancroft, and E. F. Nichols.

Committee on Program of National Research Council *for Meeting of National Academy of Sciences.*—A. O. Leuschner (Chairman), Whitman Cross, Henry M. Howe and R. M. Yerkes.

(*Approved.*)

Mr. Howe, as Chairman, submitted a report of the Committee on Organization of the Division of Industrial Relations.

*Moved:* That the report of the Committee on Organization of the Division of Industrial Relations be adopted in the following amended form:

*Organization of the Division of Industrial Relations*

It is believed that the work of this Division, particularly its first work should be largely so directed as to stimulate industrial administrators to broaden their research activities, and to persuade the smaller industries to combine for the creation and maintenance of efficient research laboratories for the common good.

The chief contribution which the Research Council has to offer the industries is the organized assistance of its Divisions of Science and Technology which are directly available for suggesting problems of industrial research and coöperating in their solution. Hence, these Divisions will serve to stimulate the industries to carry out larger and more effective schemes of coöperative research, suggested by the Division of Industrial Relations.

The Division will require a Chairman and a Vice-Chairman. The Chairman should preferably be in a position to devote his whole time to the work and should possess the highest qualifications. He must be a man who can command the attention and respectful hearing of large industries. He should have the necessary directing and executive ability combined with the power to weld together the various schemes of coöperative research.

The Division of Industrial Relations should serve as the medium through which coöperation is secured for the Divisions of Science and Technology. The actual work of organizing and directing industrial researches will be the specific work of the Divisions of Science and Technology.

The Committee therefore infers that the need of activity on the part of this Division is not so pressing now as is the case with the Divisions of Science and Technology.

The Committee recommends that the Division be constituted as follows:

*Ex Officio Members.*—The Chairmen of the Divisions of Chemistry and Chemical Technology, Engineering, and Physical Sciences.

*Representatives of Government Bureaus.*—A representative each of the Bureaus of Chemistry, Mines, and Standards.

*Members at Large.*—At least eight representatives intimately connected with industrial research or highly experienced in industrial administration. (*Adopted.*)

*Moved:* That the present Committee on Industrial Research be requested to report nominations for membership on the Division of Industrial Relations in accordance with the approved constitution of the Division. (*Adopted.*)

*Moved:* That consideration be given to changing the functions of the present Advisory Committee of the Division of General Relations so that it may serve as an Advisory Committee to the Executive Board. (*Adopted.*)

Mr. Merriam, as Chairman of the Section on Relations with Educational Institutions and State Committees, presented a report on the organization of the Division of Educational Relations.

*Moved:* That in accordance with the report submitted by the Section on Relations with Educational Institutions and State Committees, the Division of Educational Relations be organized as follows:

*Organization of the Division of Educational Relations*

It is recommended that the Research Committees in educational institutions be asked to group themselves according to six regions; that the committees in each region be requested to form an organization consisting of one or more representatives from each committee, that this organization meet for conference on the problems of Research Committees at least once each year and elect a chairman for the group; and that the Chairman of each of the six regional organizations be nominated to represent the Research Committees in the Division of Educational Relations of the National Research Council.

The following organization of the six regions is recommended:

*1. Pacific Coast Region.*—Washington, Oregon, California, Idaho, Nevada, Utah, Arizona.

*2. West Central Region.*—North Dakota, South Dakota, Nebraska, Kansas, Montana, Colorado, Wyoming.

*3. East Central Region.*—Minnesota, Iowa, Wisconsin, Michigan, Illinois, Ohio, Indiana, Kentucky.

*4. South Central Region.*—New Mexico, Texas, Oklahoma, Missouri, Arkansas, Louisiana.

*5. South Atlantic Region.*—Virginia, West Virginia, North Carolina, South Carolina, Georgia, Florida, Alabama, Tennessee, Mississippi,

*6. North Atlantic Region.*—Maine, Vermont, New Hampshire, Massachusetts, Rhode Island, Connecticut, New York, Pennsylvania, New Jersey, Delaware, Maryland, District of Columbia.

It is recommended that the membership of the Division of Educational Relations be constituted as follows:

| | |
|---|---|
| Regional representatives of research committees in educational institutions | 6 |
| Association of American Universities | 1 |
| National Association of State Universities | 1 |
| Association of American Agricultural Colleges and Experiment Stations | 1 |
| Association of American Colleges | 1 |
| American Association of University Professors | 1 |
| United States Bureau of Education | 1 |
| Divisional Nominations, not to exceed | 8 |
| | 20 |

It is recommended that the divisional nominations should represent investigators or educators of note who have given special consideration to the problem of research in educational institutions. *(Adopted.)*

Mr. Merriam, as Chairman of the Section on Relations with Educational Institutions and State Committees, presented a report on the organization of the Division of States Relations.

*Moved:* That in accordance with the report submitted by the Section on Relations with Educational Institutions and State Committees, the Division of States Relations be organized as follows:

*Organization of the Division of States Relations*

It is recommended that the State Research Committees be asked to group themselves according to six regions; that the committees in each region be requested to form an organization consisting of one or more representatives from each committee; that this organization meet for conference on the problem of state scientific organization at least once each year and elect a chairman for the group; and that the chairman of each of the six regional organizations be nominated to represent the States Committees in the Division of States Relations of the National Research Council.

The following organization of the six regions is recommended:

*1. Pacific Coast Region.*—Washington, Oregon, California, Idaho, Nevada, Utah, Arizona.

*2. West Central Region.*—North Dakota, South Dakota, Nebraska, Kansas, Montana, Colorado, Wyoming.

*3. East Central Region.*—Minnesota, Iowa, Wisconsin, Michigan, Illinois, Ohio, Indiana, Kentucky.

*4. South Central Region.*—New Mexico, Texas, Oklahoma, Missouri, Arkansas, Louisiana.

*5. South Atlantic Region.*—Virginia, West Virginia, North Carolina, South Carolina, Georgia, Florida, Alabama, Tennessee, Mississippi.

*6. North Atlantic Region.*—Maine, Vermont, New Hampshire, Massachusetts, Rhode Island, Connecticut, New York, Pennsylvania, New Jersey, Maryland, Delaware, District of Columbia.

It is recommended that the membership of the Division of States Relations be constituted as follows:

| | |
|---|---|
| Regional representatives of State Research Committees | 6 |
| Representatives of the Divisions of Science and Technology, National Research Council | 7 |
| Representatives of the Division of Educational Relations, Division of Industrial Relations, and of the Research Information Service | 3 |
| Divisional Nominations—not to exceed | 8 |
| | 24 |

(*Adopted.*)

*Moved:* That the Section on Relations with Educational Institutions and State Committees give further consideration to the apportionment of States in the regional divisions with reference to the organization of the Division of States Relations. (*Adopted.*)

Mr. Yerkes, on behalf of the Committee on Organization of the Government Division submitted the following report:

After consideration of the relations of the proposed Government Division to the other Divisions of General Relations and the possible functions of the proposed Government Division, the Committee decided to recommend that organization of the Government Division be postponed until the other Divisions of General Relations have been organized.

*Moved:* That in accordance with the recommendation of the Committee on Organization of the Government Division, the organization of the Government Division be postponed until the other Divisions of General Relations shall have been organized. (*Adopted.*)

Mr. Yerkes, Chairman of the Research Information Service, presented the following report on the organization of the Research Information Service and moved that it be approved:

*Organization of the Research Information Service*

It is recommended that the membership of the Committee include: (1) Representatives of the Divisions of the National Research Council, (2) Representatives of the Government Departments, and (3) members at large, as follows:

*Representatives from the Council.*—The Chairman of each Division.

*Representatives from Government Departments.*—One each for Agriculture, War, Navy, Commerce, Labor, Interior, Justice, State, Post Office, and Treasury.

*Members at large.*—The number not to exceed ten, representing institutions or other organizations of special importance for the Division.

The Committee would consist of thirty-three members, assuming the National Research Council representation to be thirteen, representation of Government Departments ten, and memberhip at large ten.

It is further recommended that appointments shall be so arranged that not more than one-third of the Committee membership shall retire in any one year.

The Committee would be expected to elect a Vice-chairman and to organize a small Executive Committee.

The following tentative plan of work for the Research Information Service is submitted:

1. The proper cataloguing, analysis, and filing of informational reports from various sources. (Force required two.)

2. Research personnel information. Preparation of catalogue to serve as research personnel directory. Coöperation with Editor of *American Men of Science* recommended. (Force one or two.)

3. Collection of information and investigation of ways of furthering scientific bibliographies, abstracts, and handbooks, especially analytical abstracts and handbooks. This involves survey of the present situation in coöperation with Committee on Aids to Scholarship of the Institute of International Relations. (Force two.)

4. Organization of scientific and technical research information. Establishing of contacts with informational sources. (Force one or two.) (*Adopted.*)

*Moved:* That the Interim Committee place itself on record as understanding that the Chairman of the National Research Council is ex officio a member of all committees. (*Adopted.*)

*Moved:* That Miss Ruth Cobb be appointed to assist with the work of the Research Information Service at a salary of $1800 a year, the appointment to take effect not later than May 1, 1919. (*Adopted.*)

The Chairman presented the resignation of Mr. S. J. Farnsworth, Engineering Associate to the Scientific Attaché at London, to take effect April 1.

*Moved:* That the resignation of Mr. S. J. Farnsworth as Engineering Associate to the Scientific Attaché at London, to take effect April 1, be accepted with regret, and that the Chairman express to him the appreciation of the Council for the valuable services which he has rendered. (*Adopted.*)

*Moved:* That Mr. Henry M. Howe be appointed Scientific Attaché to the American Embassy at Paris, with appointment to date from April 16, 1919. (*Adopted.*)

*Moved:* That Mr. Clevenger be appointed Acting Chairman of the Division of Engineering during the absence of Mr. Howe, the Chairman. (*Adopted.*)

Mr. Howe, Chairman of the Division of Engineering, recommended that the Division of Engineering be authorized to form a small committee to study the relation of Neumann bands in iron and steel to the rate of rupture as caused by explosions or other stresses. It was stated that this work could be

carried on with little or no expense on account of the proposed coöperation of the Explosives Section of the Bureau of Mines and other agencies.

*Moved:* That the Division of Engineering be authorized to form a committee for the study of the relation of the Neumann bands in iron and steel to the rate of rupture as caused by explosions or other stresses. (*Adopted.*)

*Moved:* That the Acting Chairman appoint a nominating committee to nominate a Chairman, a Vice Chairman, and not more than ten members at large of the Executive Board of the National Research Council to be elected at a special meeting of the Executive Board on Wednesday, April 30, at 4 p.m. (*Adopted.*)

Appointed: Messrs. Hale (Chairman), Noyes, and Millikan.

Resolutions were proposed by the Treasurer and by the Acting Secretary of the Council to interpret the following former actions taken by the Executive Board:

Joint meeting of Board with Council of National Academy, February 11, 1919:

That the scale of annual salaries of the National Research Council be as follows: Chairman of the National Research Council, $10,000; Chairmen of Divisions of Science and Technology, $6,000, with an additional allowance of $1,000 for traveling expenses if they are sent abroad by the Council; Scientific Attachés, $6,000, until the Council is in a position to pay more.

Joint Meeting of Board with Council of National Academy, March 11, 1919:

That the new Divisions be considered to be effective when the divisional nominations have been approved by the Executive Board, and that present officers of Divisions shall continue to serve until their successors shall have been elected.

Interim Committee meeting, March 18, 1919:

That the first meeting of each of the permanent Divisions, at which the Division is organized and elects its officers and executive committee, be recognized as the annual meeting of the Division for this year.

*Moved:* That the resolution of February 11 be amended by the addition of the words "It is understood that this scale of salaries is to go into effect under the new organization on July 1, 1919, provided that officers elected under the new organization prior to that time shall be entitled to the foregoing salaries from the date on which they enter upon their duties on full time subject to special action of the Executive Board." (*Adopted.*)

*Moved:* That the resolution of March 11 be amended by the insertion of the words "under present salary arrangement" between the words "serve" and "until," and by the addition of the clause "and if they are elected to act for newly elected officers until the latter assume their duties, that they shall be entitled to the salaries provided for under the new organization subject to special action of the Executive Board." (*Adopted.*)

*Moved:* That consideration of individual cases that may come under these amended resolutions be referred to the Interim Committee with power. (*Adopted.*)

Mr. Cross presented a report of progress of the Committee on Organization of Administration with the following recommendation:

*Moved:* That William M. Davies and T. C. Mooney be appointed draftsmen at a salary of $125 a month from the date of their discharge from the army until further notice up to June 30, 1919. (*Adopted.*)

Mr. Leuschner, as Chairman of the Committee on Program of the National Research Council for the annual meeting of the National Academy of Sciences submitted the following recommendation:

Since all matters pertaining to the organization and activities of the National Research Council, which require action by the Academy, will be reported by the Council of the Academy at the regular business sessions of the Academy, the committee recommends that at the open meeting of the Academy on Wednesday afternoon, April 30, Dr. Hale present an address on the past work and future plans of the National Research Council and that this address be followed by brief statements of the work of individual divisions by their chairmen and by especially invited representatives of research activities related to the war.

(*Approved.*)

The report of the Treasurer for the month ending March 31, 1919, was presented and placed on record.

Mr. Cross reported for the Budget Committee that an arrangement had been made by which disbursements for the London and Paris offices of the Research Information Service would be made through the Council of National Defense in Washington, after discontinuance of its disbursing office in London, about June 1.

Mr. Hale reported that in accordance with the resolution adopted March 11, 1919, at the joint meeting of the Executive Board with the Council of the National Academy of Sciences, authorizing a project of which the main purpose is to promote fundamental research in Physics and Chemistry in educational institutions primarily through the maintenance of a system of National Research Fellowships, a communication had been addressed by the President of the National Academy of Sciences and by the Chairman of the National Research Council to Dr. George E. Vincent, President of the Rockefeller Foundation, requesting an appropriation of $500,000, to be extended through a period of five years to enable the Council to carry out a project for promoting fundamental research in Physics and Chemistry in educational institutions in the United States through the maintenance of a system of National Research Fellowships in Physics and Chemistry and through such supplementary features as may promote the broad purpose of the project and increase its efficiency and that in reply to this request the following communication had been received from Mr. Edwin R. Embree, Secretary of the Rockefeller Foundation:

THE ROCKEFELLER FOUNDATION
61 BROADWAY, NEW YORK

April 10, 1919.

*My dear Dr. Hale:*

I have the honor to inform you that a meeting of the Executive Committee of the Rockefeller Foundation held April 9, 1919, the following resolutions were adopted:

RESOLVED that the sum of Fifty thousand dollars ($50,000) be, and it is hereby, appropriated, of which so much as may be necessary shall be paid to the NATIONAL RESEARCH COUNCIL during the year 1919 for the maintenance of a system of NATIONAL RESEARCH FELLOWSHIPS IN PHYSICS AND CHEMISTRY under the direction of the Research Board of that Council and in general conformity with the plans outlined in letter of application.

RESOLVED that the Rockefeller Foundation pledge itself to appropriate to the NATIONAL RESEARCH COUNCIL for the maintenance of a system of NATIONAL RESEARCH FELLOWSHIPS IN PHYSICS AND CHEMISTRY such additional sums for use in succeeding years as shall make available for expenditure during the period from May 1, 1919, to June 30, 1925, a total sum not to exceed Five hundred thousand dollars ($500,000), it being understood that the appropriation for any one year shall be for not more than One hundred thousand dollars ($100,000) plus any unexpended balances from appropriations for previous years.

Very truly yours,

EDWIN R. EMBREE,
*Secretary.*

*Moved:* That a committee be appointed to express to the Rockefeller Foundation the appreciation of the National Research Council for the Foundation's willingness to cooperate with the Council in the promotion of research, and of its munificence in supporting by the sum of Five Hundred Thousand Dollars the maintenance of National Research Fellowships in Physics and Chemistry.

Appointed: Messrs. Hale (Chairman), Noyes, and Millikan.

An announcement in regard to these Fellowships was issued by the National Research Council and is printed with these Minutes, page 313.

Mr. Yerkes, Chairman of the Research Information Service, presented a report of the Committee on Organization of the Government Division which was approved in the following amended form:

(1) The membership of the Government Division shall consist of the heads of such bureaus, civil and military, of Departments of the United States Government as shall be determined by the Executive Board of the National Research Council and by the Council of the National Academy of Sciences in accordance with Article V, Section I, of the Organization of the National Research Council.

(2) The Chairman of the Government Division shall be nominated by the Executive Board of the National Research Council and the President of the National Academy of Sciences shall be requested to present the name so nominated to the President of the United States for designation as Chairman of the Government Division.

(3) The Chairman of the Research Information Service shall be a member of the Government Division and shall serve as its Secretary.

The Executive Board concurred in the opinion of the committee that the Government Division has a very important function in bringing together at

intervals the heads of scientific bureaus of the Government and in promoting effective coöperation between them and also between their several bureaus and the National Research Council.

Mr. Hale presented a report on the organization of the Division of Foreign Relations and stated that in so far as has been determined at this time the representation on the Division would be as follows:

*Representing the State Department.* Hon. William Phillips, Assistant Secretary of State.
*Representing the National Academy of Sciences.*—Foreign Secretary, George E. Hale.
*Representing the American Philosophical Society.*—Henry Fairfield Osborn.
*Representing the American Academy of Arts and Sciences.*—A. A. Noyes.

Mr. Noyes reported that the Division of Chemistry and Chemical Technology had considered the report to the Executive Committee of the International Research Council from the Committee on International Coöperation in Chemistry appointed at the Paris Conference and recommended that it be concurred in by the Council with certain changes proposed by the Division.

*Moved:* That the report of the Division of Chemistry and Chemical Technology with reference to the proposed International Chemical Council be referred to the Division of Foreign Relations with the understanding that Messrs. Bancroft, Noyes, and Washburn be invited to meet with the Division for consideration of the report.

In accordance with these resolutions the following recommendations in regard to the formulation of an International Chemical Council were approved.

1. That an International Chemical Council be constituted; and that, if possible, arrangements be made for transferring the funds originally given to the International Association of Chemical Societies to this International Chemical Council.

2. That the object of the International Chemical Council be to initiate and promote international cooperation in chemistry; for example, by arranging:

(a) for international cooperation in the preparation and publication of chemical literature.

(b) for the appointment of international commissions to deal with special chemical questions of standardization (such as atomic weights, nomenclature, etc.)

(c) for international cooperation in the prosecution of special research projects.

(d) for the calling of international chemical conferences for various purposes; and also for the organization of an International Chemical Congress with meetings at stated intervals, and including all of the scientific and technological branches of chemistry.

3. That the International Chemical Council be constituted of delegates representing the leading chemical societies and other chemical research organizations of the several allied and neutral countries, these delegates to be selects as described in Paragraph 5.

4. That the International Chemical Council be affiliated with the International Research Council; and that the National Research Council of each country or its National Academy when no Research Council has been created, act as the intermediary in communications between the International Chemical Council and the chemical organizations of that country, and arrange for the proper representation of those organizations in accordance with Paragraph 5.

5. That the delegates from each country shall in general be chosen by the major chemical societies in that country, but that the number and distribution of such delegates and their voting strength within the delegation be determined initially by the National Research Council of that country, with the understanding that in countries where a National Research

Council shall not have been organized the National Academy itself shall fulfill this function until the National Research Council is organized.

6. That upon all questions voted upon by the International Chemical Council the number of votes cast by the various countries shall be determined by their population as follows:

| | |
|---|---|
| Countries of less than | 5 million inhabitants have 1 vote. |
| Countries between 5 and 10 | million inhabitants have 2 votes. |
| Countries between 10 and 15 | million inhabitants have 3 votes. |
| Countries between 15 and 20 | million inhabitants have 4 votes. |
| Countries over 20 | million inhabitants have 5 votes. |

The inhabitants of colonies and possessions are included in the population of the country to which they belong, according to the indications of its Government. Each self-governing Dominion has the same number of votes as an independent country according to the above scale.

7. That the International Chemical Council, as soon as it shall be organized, shall elect an Executive Committee of seven members, which shall exercise such functions as may be assigned to it by the Council. The Executive Committee shall appoint an Executive Secretary, who shall have charge of correspondence and of the central office of the Council.

8. That until the International Chemical Council shall be organized and its Executive Committee appointed, the Committee on International Cooperation in Chemistry appointed by the Paris Conference shall act as a provisional Executive Committee for purposes of organization. Its membership shall, however, be increased by the addition of four members representing industrial chemistry, to be appointed respectively by the Royal Society of London, the Academie des Sciences de France, the Academia dei Lincei, and the National Research Council of the United States. This Committee shall elect a chairman and a secretary; but the latter need not be a member of the Committee.

Mr. Leuschner reported that the Committee on the Enlargement of the Functions of the International Bureau of Weights and Measures, appointment of which had been previously authorized by the Interim Committee, was constituted as follows: S. W. Stratton, Chairman, Comfort A. Adams, Joseph S. Ames, Gano Dunn, Henry M. Howe, Edward B. Hyde, Albert A. Michelson, Ernest F. Nichols, Edward B. Rosa, and the Chairman of the Division of Physical Sciences—ex officio and that this Committee had submitted the following report:

WHEREAS, practically all scientific investigations and much technological work required uniformity in fundamental standards of measurement, values of physical constants, and methods of measurement; and

WHEREAS, there is at present no provision for bringing about this uniformity other than that of the Treaty establishing the International Bureau of Weights and Measures,—charged with matters pertaining to length and mass only.

IT IS RECOMMENDED,—that the Treaty providing for the establishing and maintenance of the International Bureau of Weights and Measures be amended to provide for the following functions:

1. To serve as a depository for such standards as require the preservation of prototypes at a central place, and to make comparisons between these prototypes and the national prototypes of countries subscribing to the Treaty.

2. To establish international values of constants by the correlation of data produced at the various national and other scientific laboratories, such as the mechanical equivalent of heat, melting points, boiling points, gravitation constant, fundamental electrical units, velocity of light, etc.

3. To undertake investigations concerning standards, constants, or methods of measurement, which from their nature must be undertaken jointly by the nations subscribing to the Treaty.

4. To correlate and utilize, as far as possible, the work of the various national and other laboratories pertaining to standards, or methods of measurement, and to promulgate the results.

5. To provide for an increase in the membership of the Committee charged with the direction of the work of the International Bureau in order to secure a group of experts representative of the wider range of subjects.

*Moved:* That the Executive Board of the National Research Council express approval of the principle contained in the suggestion of the Executive Committee of the Division of Physical Sciences, that the scope of the International Bureau of Weights and Measures be enlarged, and that steps be taken, in consultation with national and international bodies interested, to determine the constants to be included, with a view to enlarging the function of the Bureau in harmony with the existing work and jurisdiction of other international bodies. (*Adopted.*)

On behalf of the Committee on Enlargement of the Functions of the International Bureau of Weights and Measures, Mr. Leuschner recommended that Messrs. Comfort A Adams, Joseph S. Ames, and A. A. Michelson be appointed as a special committee to report on a wave length standard for a unit of length. (*Approved.*)

Mr. Leuschner, Acting Chairman of the Division of Physical Sciences, submitted the following recommendations:

1. That Messrs. William Bowie, F. R. Moulton and C. F. Marvin be appointed a Committee on Variation of Latitude of the American Section of the International Geophysical Union, to confer with a similar committee of the American Section of the International Astronomical Union and to make joint recommendations with this Committee in regard to the future organization of researches on the variation of latitude.

2. That Messrs. L. A. Bauer, William Bowie, Whitman Cross, H. F. Reid, C. F. Marvin, A. O. Leuschner, and R. S. Woodward (Chairman), be appointed a committee to prepare recommendations regarding international cooperation in geophysical subjects for consideration by the American Section of the International Geophysical Union, with the understanding that this Committee has power to increase its membership.

3. That Mr. William Bowie be appointed Acting Chairman of the American Section of the International Geophysical Union.

4. That the organization meeting of the American Section of the Geophysical Union be held in Washington in conjunction with the June meeting of the American Section of the International Astronomical Union.

*Moved:* That the foregoing recommendations be adopted and that the nominations be approved, and that the further organization of the American Section of the International Geophysical Union be left with its Acting Chairman with power. (*Adopted.*)

On motion of Mr. Howe, the Committee on Organization of the Division of Industrial Relations, was discharged.

Mr. Howe presented a communication from Mr. Alfred D. Flinn, Secretary of Engineering Foundation, transmitting a report of the Foundation's Committee on Relations with the National Research Council. Mr. Dunn directed attention to certain changes in the report concerning the membership of the Division agreed upon by the Engineering Division at its meeting in New York on April 12. The report as amended is as follows:

ENGINEERING FOUNDATION

*Report of Committee on Relations with National Research Council*

W. F. M. Goss, Chairman, Charles F. Rand, Silas H. Woodard, Frank B. Jewett Appointed at Meeting of Board, February 13, 1919.

April 10, 1919.

*To Engineering Foundation Board:*

Your Committee on Relations with National Research Council, after several meetings and conferences, recommends the approval by the Foundation Board of the following proposals:

1. Engineering Foundation, recognizing the desirability of maintaining close affiliation with National Research Council, proposes to collaborate with the Council "for the furtherance of research in science and engineering or for the advancement in any other manner of the profession of engineering and the good of mankind."

2. To contribute to the above end, as part of the policy of Engineering Foundation, office space in Engineering Societies Building has been engaged at the expense of Engineering Foundation, in addition to its own requirements, to serve as the New York office of the Engineering Division of the National Research Council, beginning May 1, 1919, and the Foundation, having brought its office to an adjacent room, in addition proffers to the Council and its Engineering Division, without charge, such secretarial services as the Foundation may from time to time determine.

3. National Research Council has proposed that its Engineering Division, comprising in all not less than 24 nor more than 29 members (of whom at least 7 and not more than 12 shall be members at large), be so organized as to include at least 5 members of Engineering Foundation, and (including these 5) 17 members of the Founder Societies. Engineering Foundation accepts this proposal as well calculated to meet the mutual requirements of the Foundation and the National Research Council.

4. Engineering Foundation proposes to collaborate with National Research Council in the activities of its Engineering Division and to make such appropriations of funds to aid specific undertakings of the Division as the Foundation may from time to time determine.

5. It is understood that all publications relating to research work in which the Foundation shall have participated, will be issued under the joint names of the Engineering Foundation and the National Research Council.

Respectfully,
W. F. M. Goss.
*Chairman.*

*Moved:* That the policy recommended by the Engineering Foundation's Committee on Relations with the National Research Council in its report of April 10, 1919, is acceptable to the National Research Council and will receive its approval if adopted by the Foundation, and that in anticipation of the adoption of the report by the Engineering Foundation the National Research Council express its appreciation of the Foundation's cooperation with the Council in the promotion of engineering research. (*Adopted.*)

Mr. Howe presented a recommendation of the Engineering Division that the number of members at large be increased from seven to twelve.

(*Approved.*)

*Moved:* That the Engineering Division move its offices to New York at as early a date as possible after May 1, 1919. (*Adopted.*)

*Moved:* That the material belonging to the National Research Council, with the exception of letter files and drawings now located in the shops of the Carnegie Institute of Technology at Pittsburgh, be turned over to that Institute with the request that they act as custodian of it until called for by the National Research Council. (*Adopted.*)

On behalf of the Division of Engineering, Mr. Howe offered the services of the Division to the National Research Fellowship Board in Physics and Chemistry.

On behalf of the Executive Committee of the Division of Medical Sciences, Mr. Hussey recommended that a grant of $800 from the funds of the Division be made to support the work of making an analysis of the psychological records of medical officers taken during the period of mobilization.

(*Approved.*)

Mr. Hale presented his resignation as Chairman of the National Research Council to take effect April 30, 1919.

The resignation of the Chairman was accepted with regret and the following resolution was unanimously adopted:

RESOLVED: That as a mark of appreciation of the constructive services, characterized by accomplishment as well as vision, rendered in the early organization and in the past years chairmanship of the National Research Council by George Ellery Hale the Council in accepting the resignation of the Chairmanship he has just presented hereby creates and bestows upon him in perpetuity the title of Honorary Chairman of the National Research Council and requests the presiding officer to appoint a committee of three to prepare and inscribe in the records a suitable minute more fully setting forth the services rendered and more fully expressing the sense of obligation of the National Research Council.

The Chairman appointed Messrs. Dunn (Chairman), Noyes, and Walcott.

The Meeting adjourned at 1 p.m.

PAUL BROCKETT, *Assistant Secretary.*

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## NATIONAL RESEARCH FELLOWSHIPS IN PHYSICS AND CHEMISTRY

SUPPORTED BY THE ROCKEFELLER FOUNDATION

*General Statement.*—The National Research Council has been entrusted by the Rockefeller Foundation with the expenditure of an appropriation of $500,000 within a period of five years for promoting fundamental research in physics and chemistry primarily in educational institutions of the United States.

The primary feature of the plan is the initiation and maintenance of a system of National Research Fellowships, which are to be awarded by the National Research Council to persons who have demonstrated a high order of ability in research, for the purpose of enabling them to conduct investigations at educational institutions which make adequate provision for effective prosecution of research in physics or chemistry. The plan will include such supplementary features as may promote its broad purpose and increase its efficiency.

*Purposes in View.* Among the important results which are expected to follow from the execution of the plan may be mentioned:

(1) Opening of a scientific career to a larger number of able investigators and their more thorough training in research, thus meeting an urgent need of our universities and industries.

(2) Increase of knowledge relating to the fundamental principles of physics and chemistry, upon which the progress of all the sciences and the development of industry depend.

(3) Creation of more favorable conditions for research in the educational institutions of this country.

*Administration.*—The plan will be administered by the Research Fellowship Board of the National Research Council. This Board consists of six members appointed for terms of five years, and of the chairmen ex officiis of the Division of Physical Science and the Division of Chemistry and Chemical Technology of the National Research Council. The members of the Board are

HENRY A. BUMSTEAD, Professor of Physics, Yale University.
SIMON FLEXNER, Director of Laboratories, Rockefeller Institution for Medical Research.
GEORGE E. HALE, Director of Mount Wilson Observatory.
ELMER P. KOHLER, Professor of Chemistry, Harvard University.
ROBERT A. MILLIKAN, Professor of Physics, University of Chicago.
ARTHUR A. NOYES, Director of the Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology.
WILDER D. BANCROFT, Professor of Physical Chemistry, Cornell University.
Chairman of the Division of Chemistry and Chemical Technology.
———— — ————, Chairman of the Division of Physical Sciences.

*Coöperation of Educational Institutions.*—National Research Fellows will be permitted to conduct their investigations at institutions that will coöperate in meeting their needs. These needs differ widely from those of students seeking only instruction. Able investigators, actively engaged in productive research, are needed to inspire and guide the work of the Fellows. Research laboratories, adequately manned with assistants and mechanicians, and amply supplied with instruments, machine tools, and other facilities, are indispensable; and funds to provide supplies and to satisfy the constantly recurrent demands of research must be available. Above all, there must exist the stimulating atmosphere found only in institutions that have brought together a group of men devoted to the advancement of science through pursuit of research.

The Research Fellowship Board expects to make arrangements by which educational institutions will associate the Research Fellows with their graduate departments and offer the most favorable conditions for the prosecution of their researches.

The applicant will indicate one or more institutions at which, in his opinion, his research work can be conducted to the best advantage.

*Fellowship Appointments.*—The appointments of National Research Fellows will be made only after careful consideration of the scientific attainments of all candidates, not only of those who apply on their own initiative, but also of those who are brought to the attention of the Fellowship Board by professors in educational institutions and by other investigators throughout the country. In making the appointments much weight will also be given to the

judgment shown by the applicant in selecting and planning his proposed research.

The Research Fellowships will for the most part be awarded to American citizens who have had training equivalent to that represented by the Doctor's degree. The salary will ordinarily be $1,500 for the first year. The Research Fellowship Board will not, however, be bound by rigid rules of procedure. Thus it may offer larger salaries to those of exceptional attainment or wider experience, and may give appointment to competent investigators who have had training other than that represented by the Doctor's degree. The Research Fellows will be appointed for one year; but they will be eligible for successive reappointments, ordinarily with increase of salary.

*Fellowship Regulations.*—Research Fellows are expected to devote their entire time to research, except that during the college year they may at their option give not more than one-fifth of their time (outside preparation included) to teaching of educational value to themselves, or to attendance on advanced courses of study. They may associate graduate students with their researches. They shall not engage in work for remuneration during the term of their appointment. Fellows who have not received the Doctor's degree may, with the approval of the institution, offer their research work in partial fulfillment of the requirements for that degree.

Fellows are expected to submit to the Board shortly before the first of April of each year a detailed report on the progress of their researches. They must also present an account of their researches in form for publication before withdrawing from the Fellowship; and final salary payments will be deferred until this condition is fulfilled. It is understood that all results of investigation by the Fellows shall be made available to the public without restriction.

Fellowship appointments are subject to the condition that after they are accepted by the applicant, they will not be vacated within the year without consent of the Research Fellowship Board.

*Fellowship Applications.*—It is expected that fifteen to twenty Research Fellowships will be available during the coming year, and that the number will be increased in subsequent years. Applications for these Fellowships should be made on the form provided for the purpose, and should be sent to the Secretary of the Research Fellowship Board, National Research Council, 1023 Sixteenth Street, Washington, D. C., to whom all other correspondence should also be addressed. Applications will be received up to September 1, 1919 for Fellowships available during the next academic year; but a limited number of appointments will be made on the basis of the applications received. before April 20, 1919.

*Washington, D. C., March 29, 1919.*

## ORGANIZATION OF DIVISIONS OF SCIENCE AND TECHNOLOGY

### DIVISION OF PHYSICAL SCIENCES

C. E. Mendenhall, *Chairman.*
A. O. Leuschner, *Acting Chairman.*

*Executive Committee*

C. E. Mendenhall, *Chairman.*
William Bowie.
A. O. Leuschner, *Acting Chairman.*
R. A. Millikan.
H. N. Russell.
E. B. Wilson.

*Representatives of Societies*

| | *Term of Office Years* |
|---|---|
| *American Astronomical Society* | |
| W. W. Campbell | 2 |
| H. N. Russell | 1 |
| Joel Stebbins | 3 |
| *American Physical Society* | |
| H. A. Bumstead | 1 |
| William Duane | 2 |
| Irving Langmuir | 2 |
| Ernest Merritt | 3 |
| R. A. Millikan | 1 |
| E. B. Wilson | 3 |
| *American Mathematical Society* | |
| E. W. Brown | 1 |
| L. E. Dickson | 3 |
| H. S. White | 2 |

*Members at Large*

| | |
|---|---|
| J. S. Ames | 2 |
| L. A. Bauer | 3 |
| William Bowie | 2 |
| Henry Crew | 1 |
| C. F. Marvin | 1 |
| Max Mason | 3 |
| M. I. Pupin | 1 |
| S. W. Stratton | 2 |
| A. Trowbridge | 3 |

### DIVISION OF ENGINEERING

Henry M. Howe, *Chairman.*
Galen H. Clevenger, *Vice-Chairman.*

*Executive Committee*

Henry M. Howe, *Chairman.*
Comfort A. Adams.
Galen H. Clevenger, *Vice-Chairman.*
D. S. Jacobus.
E. G. Spilsbury.

*Representatives of Societies*

| | Term of Office Years |
|---|---|
| *American Society of Mechanical Engineers* | |
| Arthur M. Greene | 3 |
| W. F. M. Goss | 1 |
| D. S. Jacobus | 2 |
| *American Institute of Electrical Engineers* | |
| Comfort A. Adams | 2 |
| F. B. Jewett | 3 |
| W. R. Whitney | 1 |
| *American Institute of Mining Engineers* | |
| Hennen Jennings | 2 |
| Philip N. Moore | 1 |
| Joseph W. Richards | 3 |
| *American Society of Civil Engineers* | |
| Anson Marston | 3 |
| H. H. Porter | 1 |
| George S. Webster | 2 |
| *American Society for Testing Materials* | |
| A. A. Stevenson | 1 |
| *American Society of Illuminating Engineers* | |
| Edward P. Hyde | 2 |
| *Western Society of Engineers* | |
| Arthur N. Talbot | 3 |
| *Society of Automotive Engineers* | |
| Charles F. Kettering | 1 |

*Members at Large*

| | |
|---|---|
| Henry M. Howe, *Chairman* | 1 |
| Galen H. Clevenger, *Vice-Chairman* | 2 |
| Edward Dean Adams | 3 |
| John J. Carty | 2 |
| Gano Dunn | 2 |
| Van H. Manning | 2 |
| Charles F. Rand | 1 |
| E. G. Spilsbury | 1 |
| Bradley Stoughton | 3 |
| S. W. Stratton | 3 |
| Ambrose Swasey | 1 |
| William H. Walker | 3 |

DIVISION OF CHEMISTRY AND CHEMICAL TECHNOLOGY

E. W. Washburn, *Acting Chairman.*
W. D. Bancroft, *Chairman.*
Julius Stieglitz, *Vice Chairman.*

*Executive Committee*

E. W. Washburn, *Acting Chairman.*
W. D. Bancroft, *Chairman-elect.*
Julius Stieglitz, *Vice Chairman.*
A. B. Lamb.
A. A. Noyes.
C. L. Alsberg.

*Representatives of Societies*

| | Term of Office Years |
|---|---|
| *American Chemical Society.* | |
| C. L. Alsberg | 1 |
| W. D. Bancroft, *Chairman-elect* | 1 |
| C. G. Derick | 1 |
| J. M. Francis | 3 |
| E. C. Franklin | 2 |
| W. F. Hillebrand | 3 |
| John Johnston | 2 |
| Julius Stieglitz, *Vice Chairman* | 3 |
| J. E. Teeple | 2 |
| *American Electrochemical Society* | |
| Colin G. Fink | 3 |
| *American Institute of Chemical Engineers* | |
| Hugh K. Moore | 1 |
| *American Ceramic Society* | |
| Albert V. Bleininger | 2 |

*Members at Large*

| | |
|---|---|
| C. H. Herty | 3 |
| G. A. Hulett | 3 |
| A. B. Lamb | 2 |
| A. A. Noyes | 1 |
| C. L. Parsons | 2 |
| E. W. Washburn | 1 |

DIVISION OF GEOLOGY AND GEOGRAPHY

E. B. Mathews, *Vice Chairman.*

*Executive Committee*

E. B. Mathews, *Vice Chairman.*
Isaiah Bowman.
A. H. Brooks.
J. M. Clarke.
N. M. Fenneman.
David White.

*Representatives of Societies*

| | Term of Office Years |
|---|---|
| *Association of American Geographers* | |
| W. M. Davis | 2 |
| N. M. Fenneman | 3 |
| J. Russell Smith | 1 |
| *American Geographical Society* | |
| Isaiah Bowman | 2 |
| *Geological Society of America* | |
| J. M. Clarke | 2 |
| Whitman Cross | 3 |
| R. A. Daly | 2 |
| H. E. Gregory | 1 |
| A. C. Lawson | 3 |
| C. K. Leith | 1 |

*Paleontological Society*
T. Wayland Vaughan ..... 1
*National Geographic Society*
Gilbert Grosvenor ..... 3

*Members at Large*

Ralph Arnold ..... 2
Eliot Blackwelder ..... 3
A. H. Brooks ..... 2
A. L. Day ..... 3
Ellsworth Hungtington ..... 2
Douglas Johnson ..... 1
E. B. Mathews, *Vice Chairman* ..... 3
R. A. F. Penrose, Jr. ..... 1
David White ..... 1

DIVISION OF BIOLOGY AND AGRICULTURE

C. E. McClung, *Chairman.*
L. R. Jones, *Vice Chairman.*

*Executive Committee.*

C. E. McClung, *Chairman.*
L. R. Jones, *Vice Chairman.*
I. W. Bailey.
F. R. Lillie.
G. R. Lyman.
H. F. Moore.
A. F. Woods.

*Representatives of Societies*

*Term of Office Years*

*American Society of Agronomy*
Charles V. Piper ..... 3
*American Society of Bacteriologists*
Samuel C. Prescott ..... 1
*Botanical Society of America*
William Crocker ..... 2
A. S. Hitchcock ..... 1
L. R. Jones ..... 3
*Ecological Society of America*
W. M. Wheeler ..... 2
*American Society of Economic Entomologists*
P. J. Parrott ..... 3
*Society of American Foresters*
Barrington Moore ..... 3
*American Genetics Association*
G. N. Collins ..... 1
*American Society for Horticultural Science*
U. P. Hedrick ..... 1
*American Phytopathological Society*
George R. Lyman ..... 2

*Society of American Zoologists*

M. F. Guyer..... 2
F. R. Lillie..... 1
G. H. Parker..... 3

*Members at Large*

I. W. Bailey..... 1
B. E. Livingston..... 3
C. E. McClung, *Chairman*..... 1
C. F. Marbut..... 3
A. G. Mayor..... 2
H. F. Moore..... 2
J. R. Murlin..... 3
W. H. Osgood..... 2
A. F. Woods..... 1

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## *EDITORIAL NOTE*

With the next issue (August, 1919) the PROCEEDINGS will return to the larger type which was initially adopted but which was reduced a year and a half ago as a War economy. The return would have been made sooner but for the large amount of matter already set up and delayed in publication by a series of unavoidable accidents.